

Garter

Garbage Collection

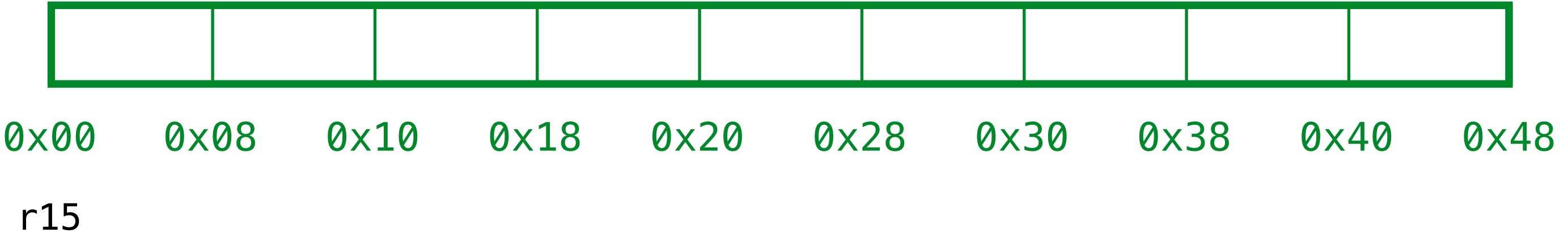
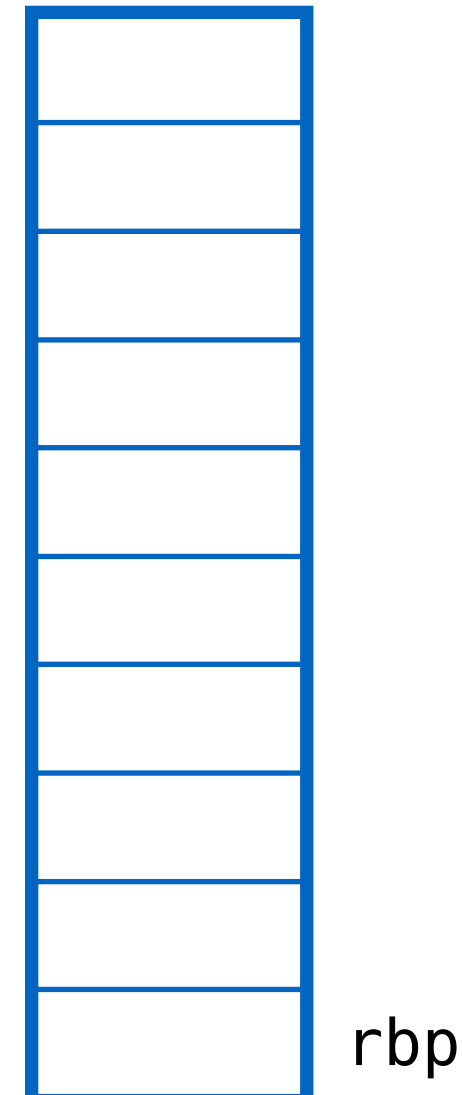
Ranjit Jhala | UCSD

Garter / GC

Example 1

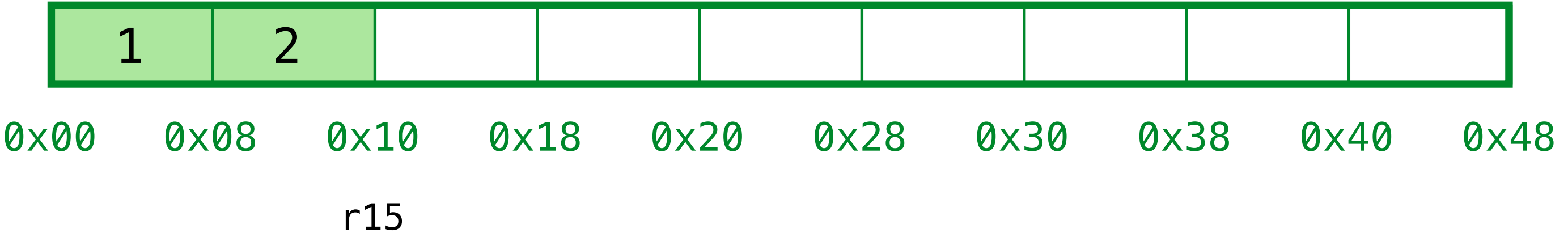
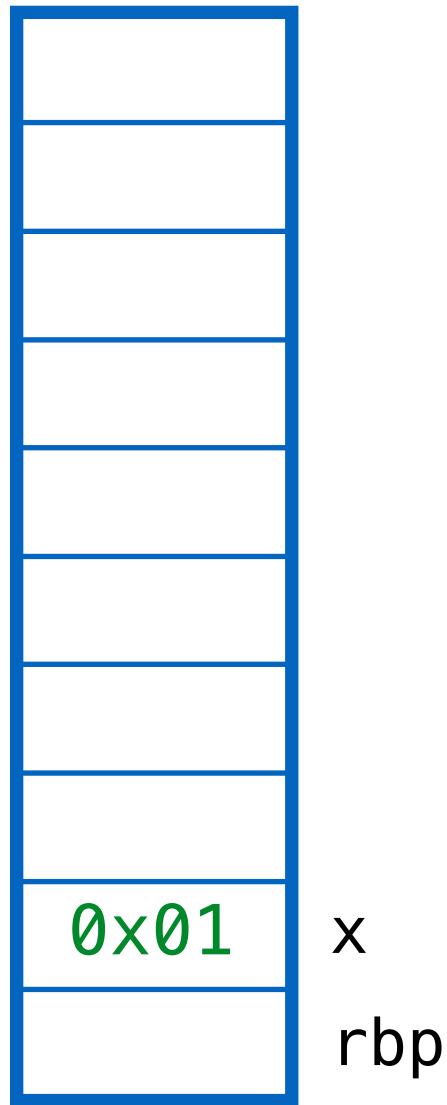
ex1: garbage at end

```
let x = (1, 2)
  , y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
  , p0 = x[0] + y
  , p1 = x[1] + y
in
(p0, p1)
```



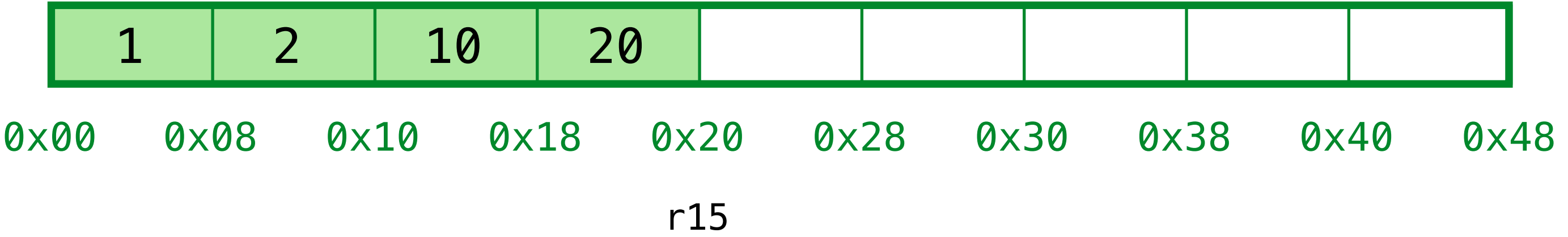
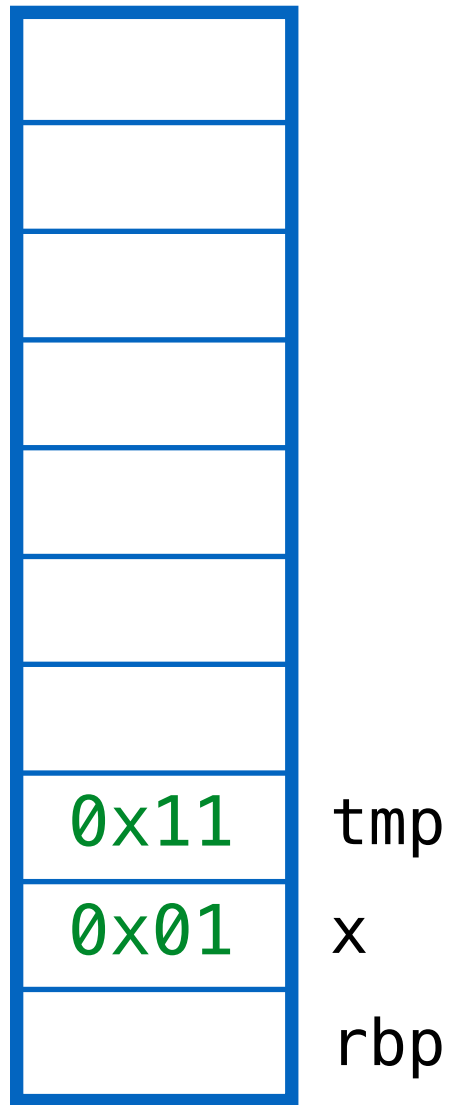
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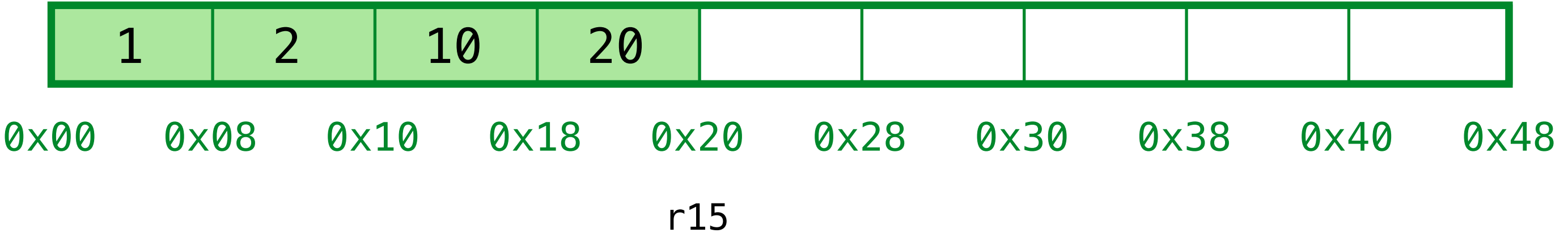
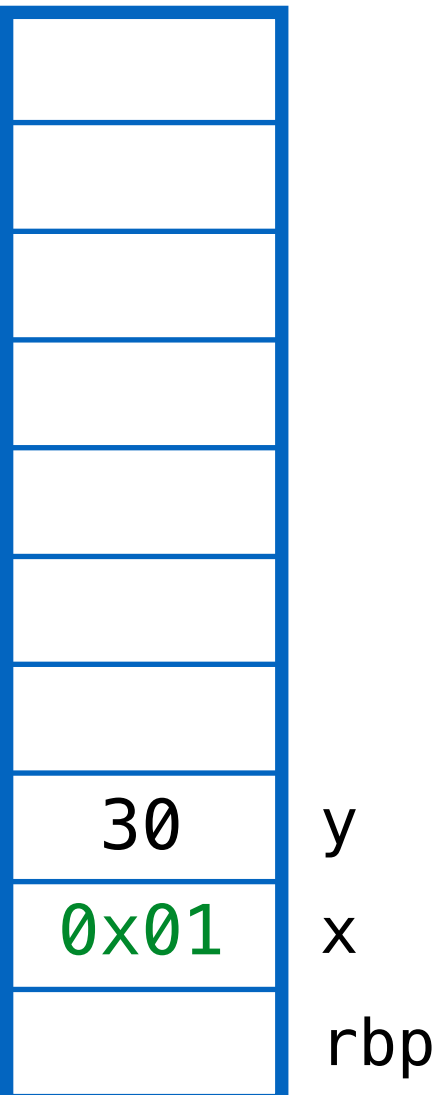
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(p0, p1)
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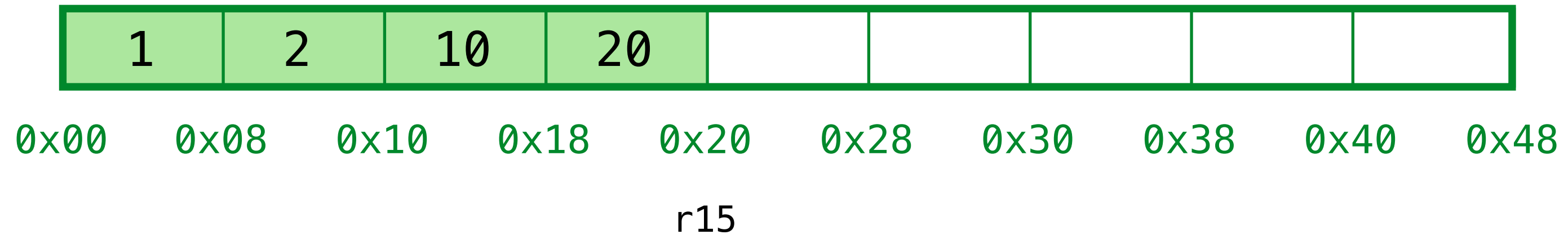
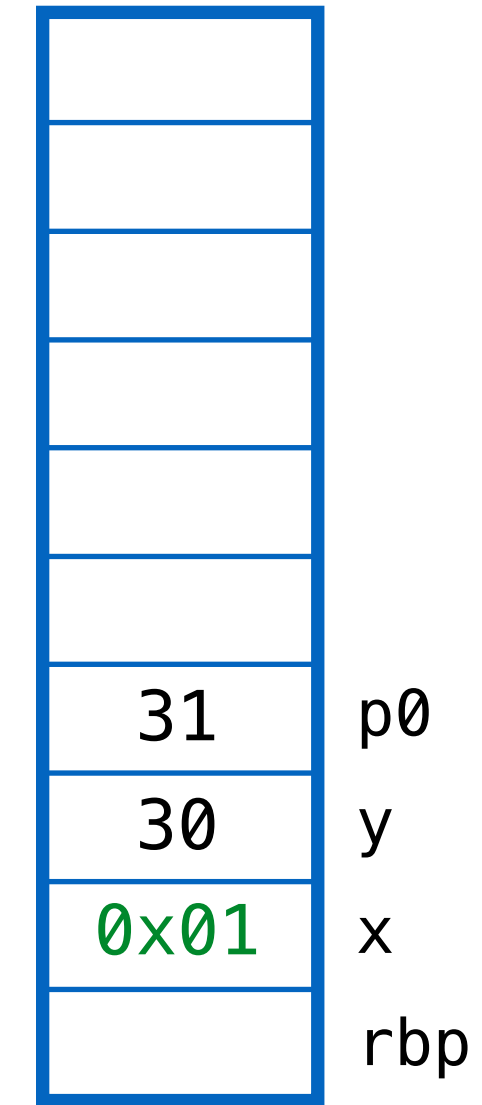
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(p0, p1)
```



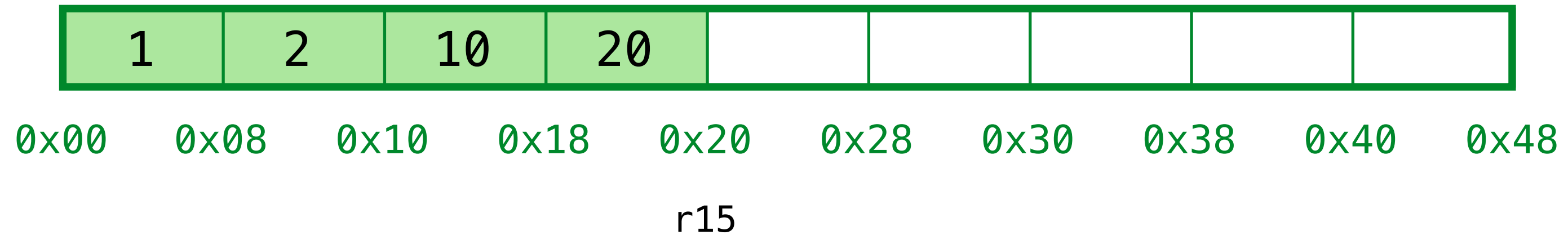
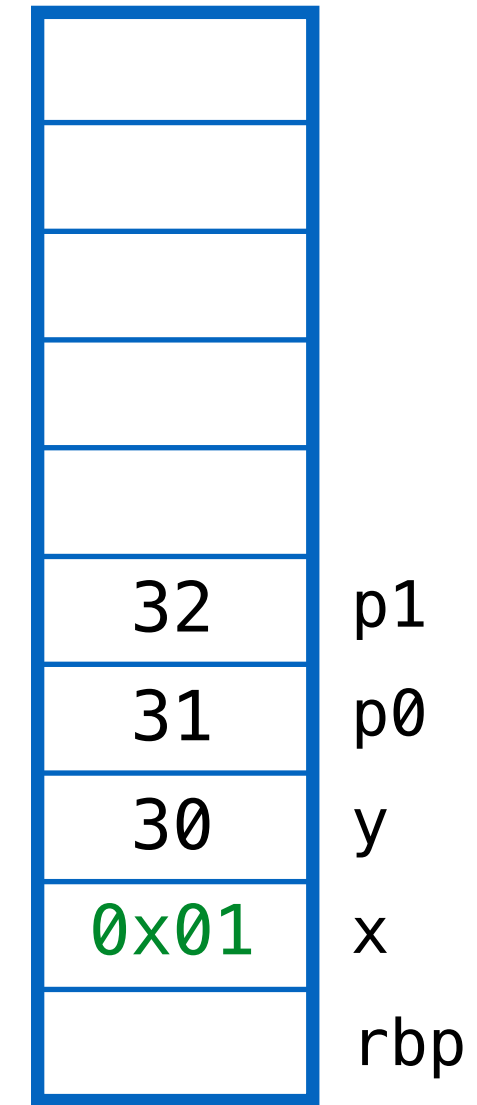
ex1: garbage at end

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let x = (1, 2)
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    , p0 = x[0] + y
    , p1 = x[1] + y
in
  (p0, p1)
```



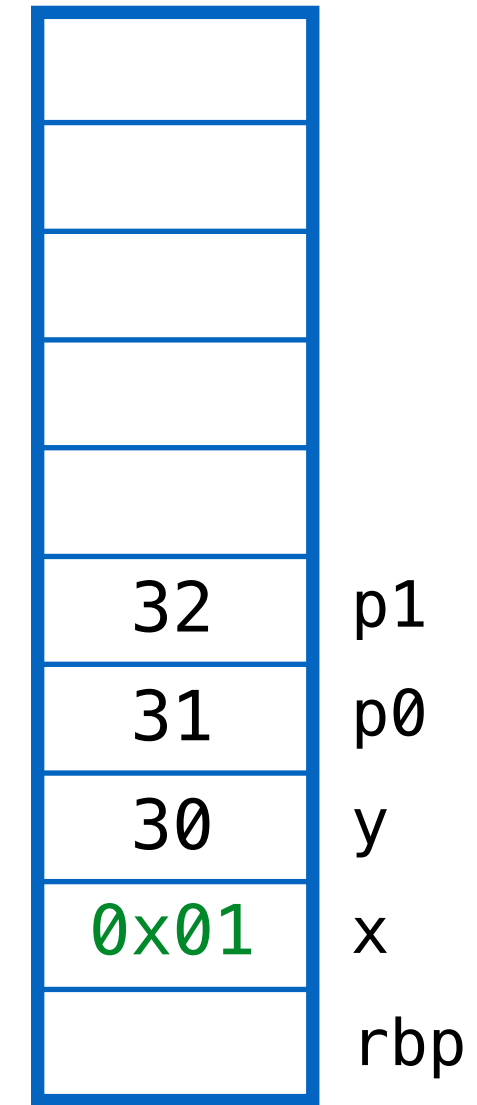
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    , p1 = x[1] + y
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```

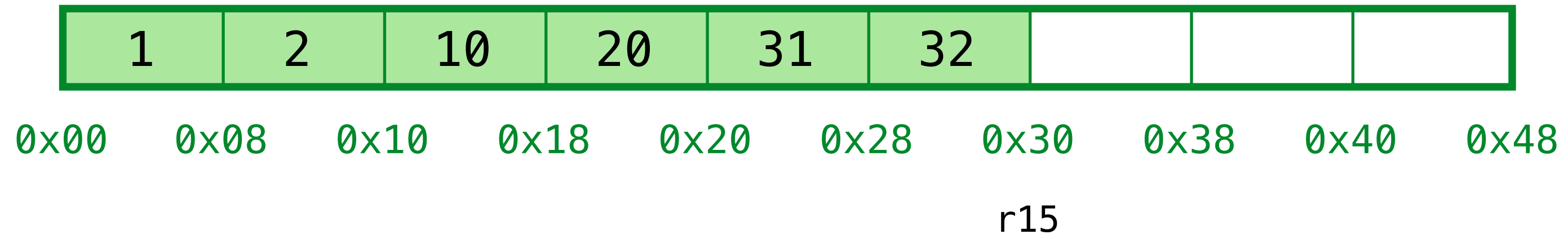


ex1: garbage at end

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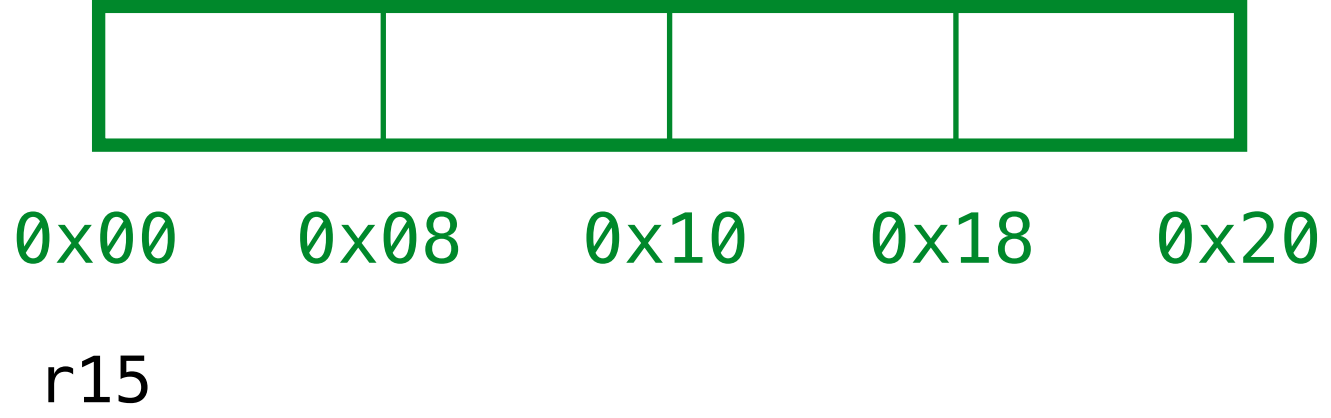
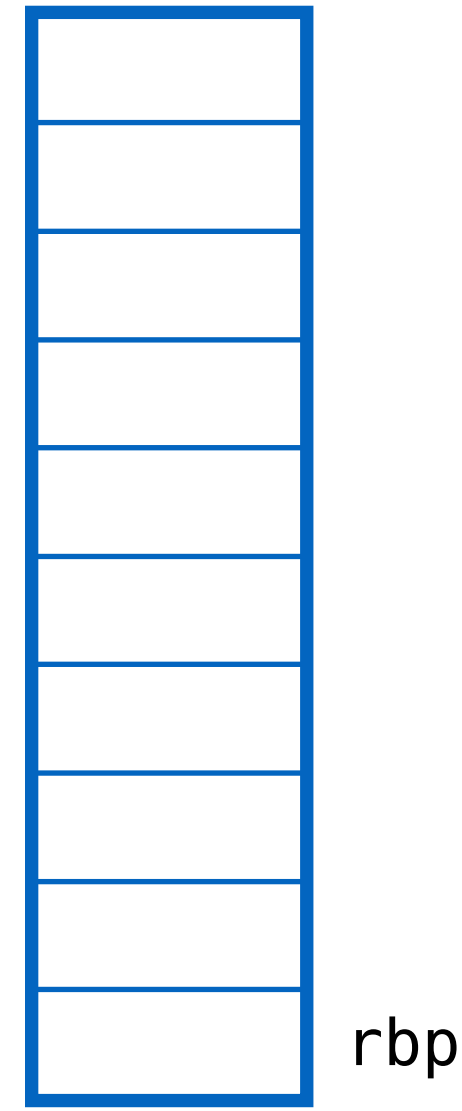


Result (rax) = 0x21



ex1: garbage at end

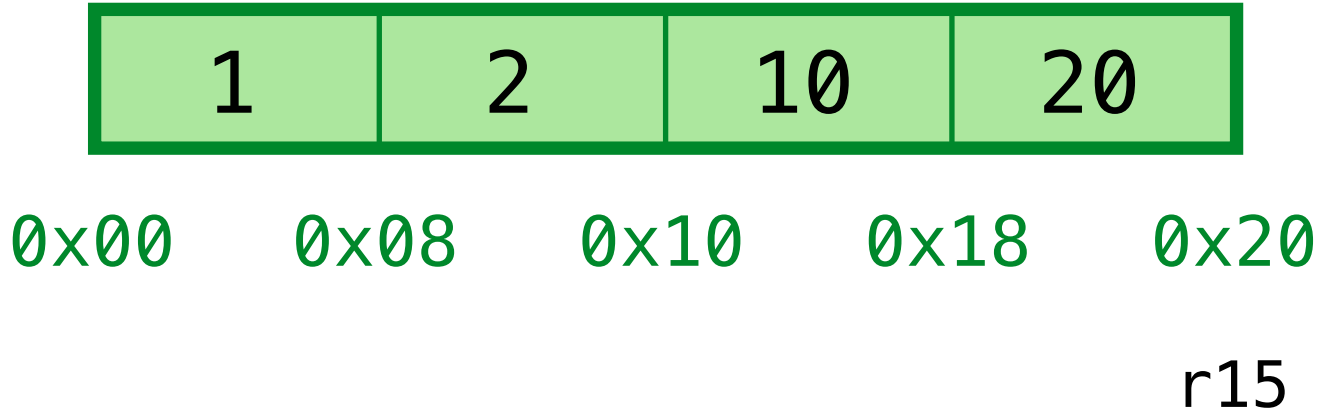
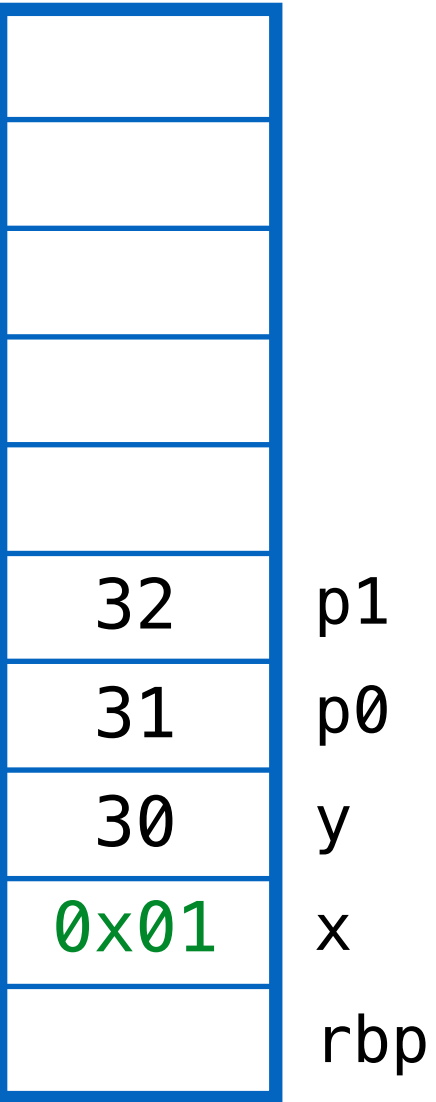
```
let x = (1, 2)
  , y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
  , p0 = x[0] + y
  , p1 = x[1] + y
in
(p0, p1)
```



Suppose we had a smaller, 4-word heap

ex1: garbage at end

```
let x = (1, 2)
  , y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
  , p0 = x[0] + y
  , p1 = x[1] + y
in (p0, p1)
```

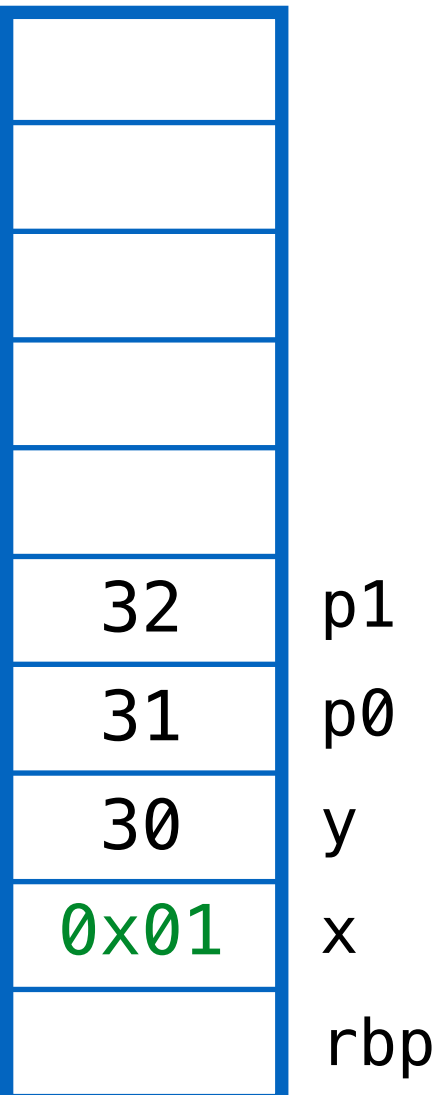


ex1: garbage at end

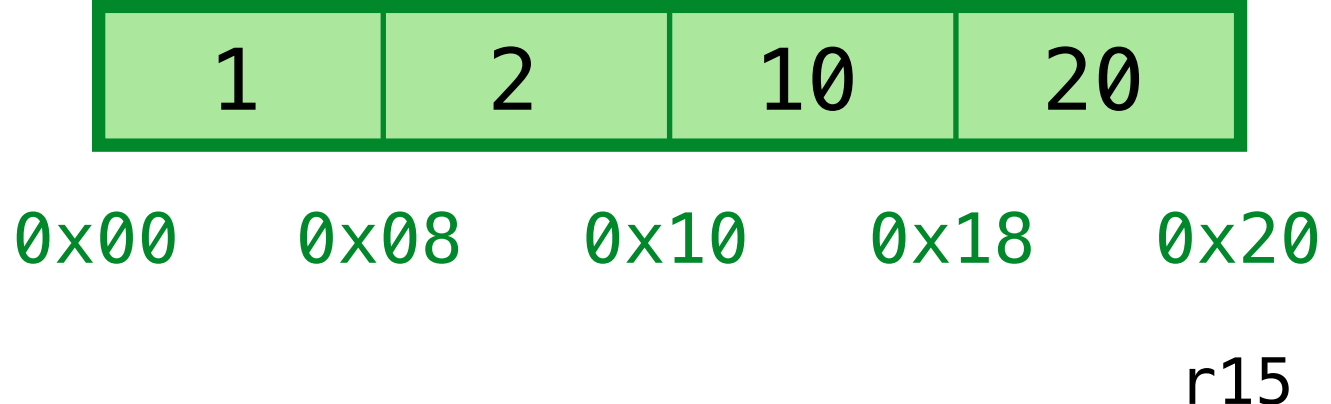
```

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    , y = let tmp = (10, 20)
          in tmp[0] + tmp[1]
    , p0 = x[0] + y
    , p1 = x[1] + y
in (p0, p1)

```

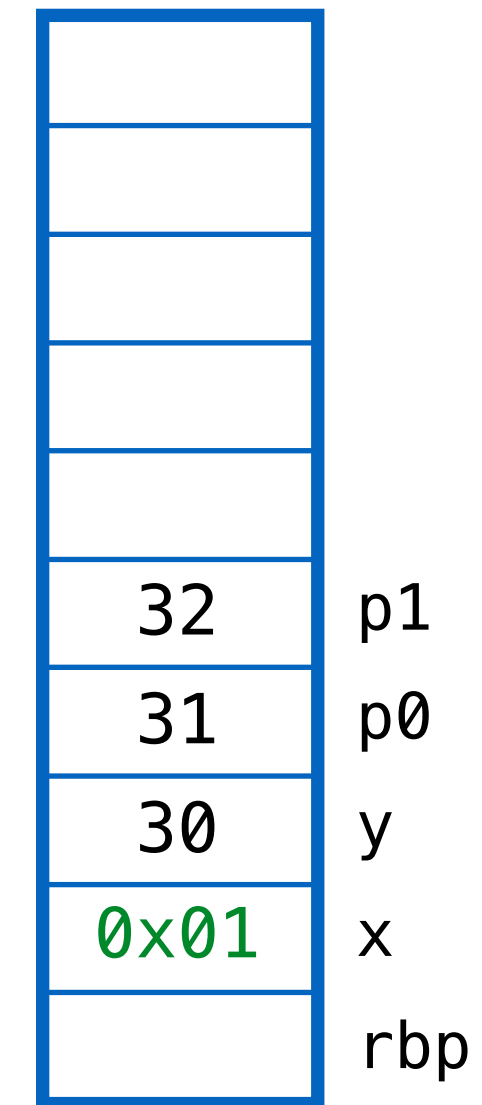


Out of memory!
Can't allocate (p0, p1)

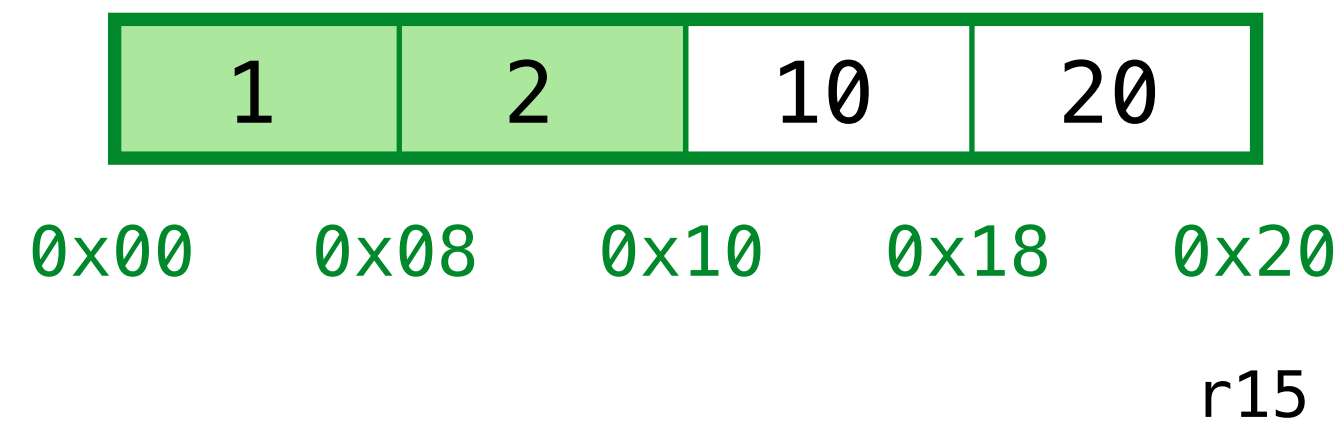


ex1: garbage at end

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          in tmp[0] + tmp[1]
    , p0 = x[0] + y
    , p1 = x[1] + y
in (p0, p1)
```



(10, 20) is “garbage”



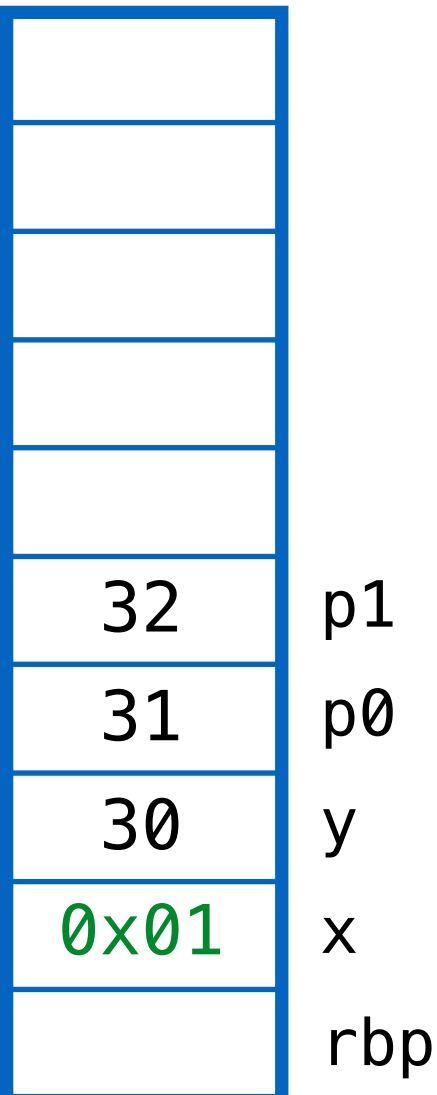
Q: How to determine if cell is garbage?

ex1: garbage at end

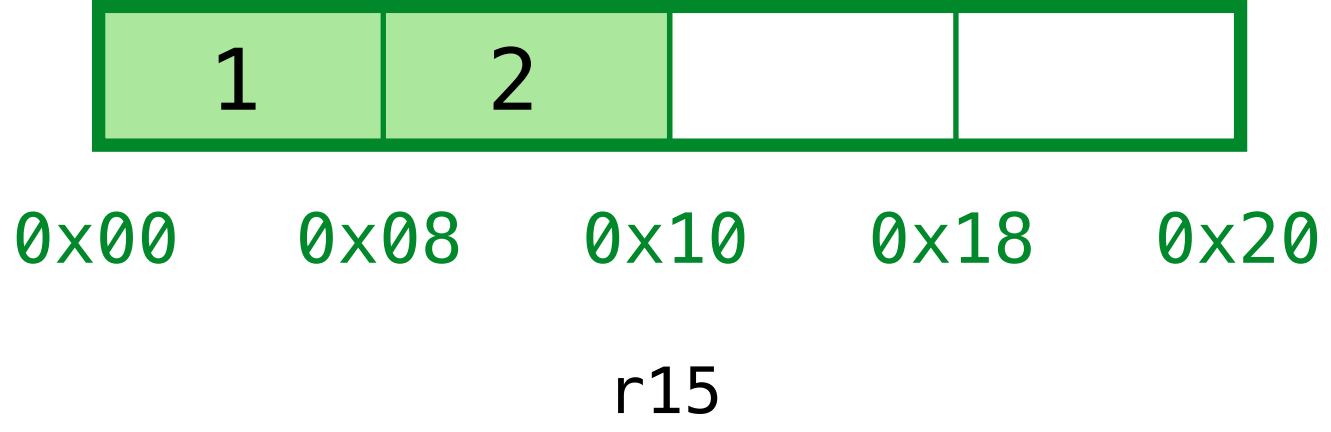
```

let x = (1, 2)
    , y = let tmp = (10, 20)
          in tmp[0] + tmp[1]
    , p0 = x[0] + y
    , p1 = x[1] + y
in (p0, p1)

```

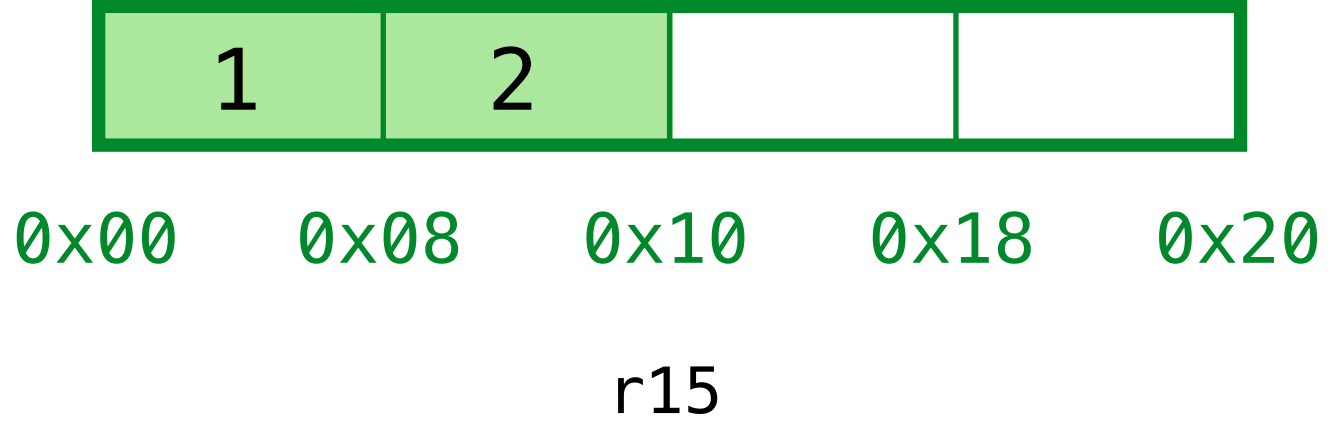
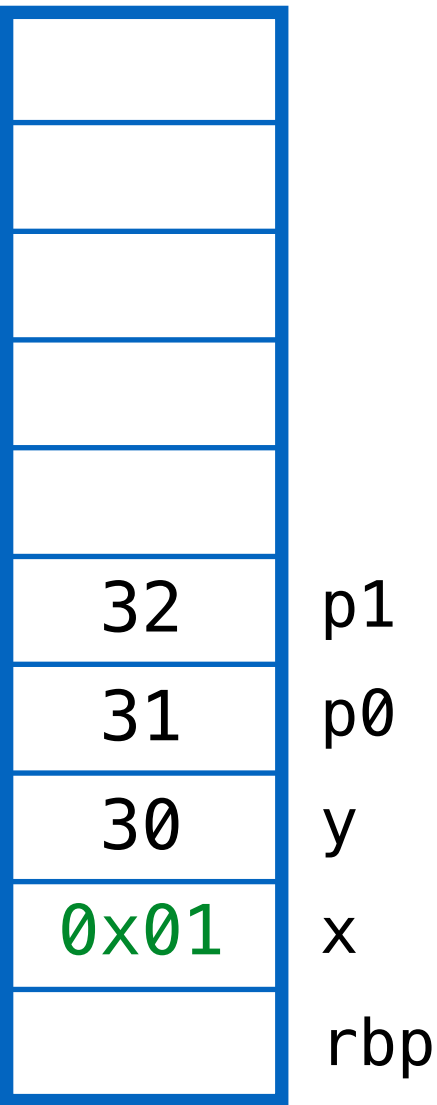


(10, 20) is "garbage"



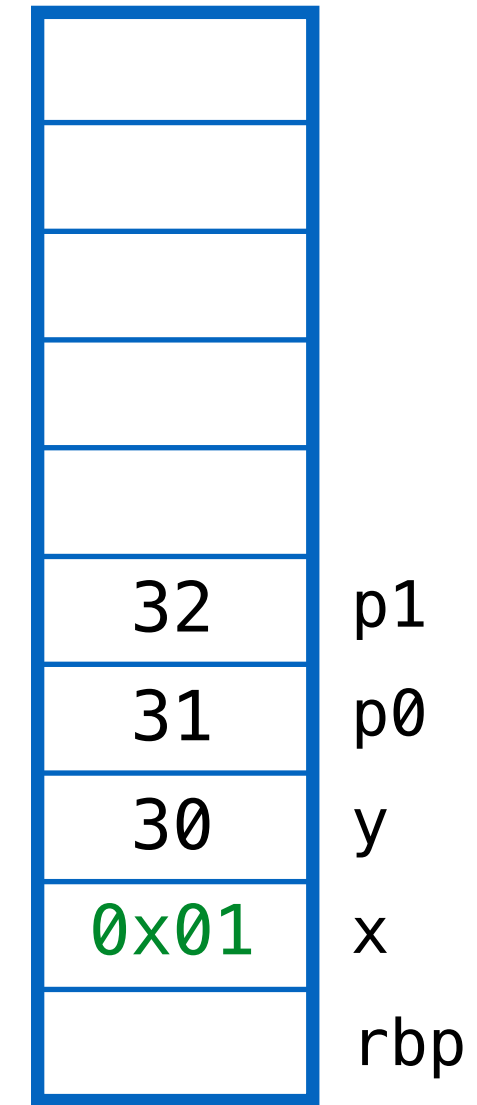
ex1: garbage at end

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        in tmp[0] + tmp[1]
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in (p0, p1)
```

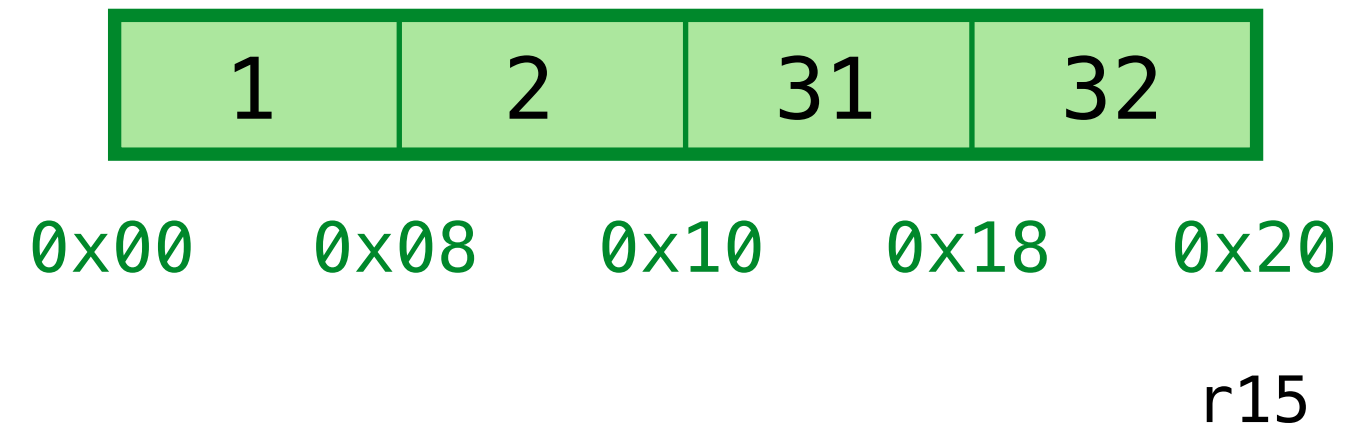


ex1: garbage at end

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in
(p0, p1)
```



Result (rax) = 0x11

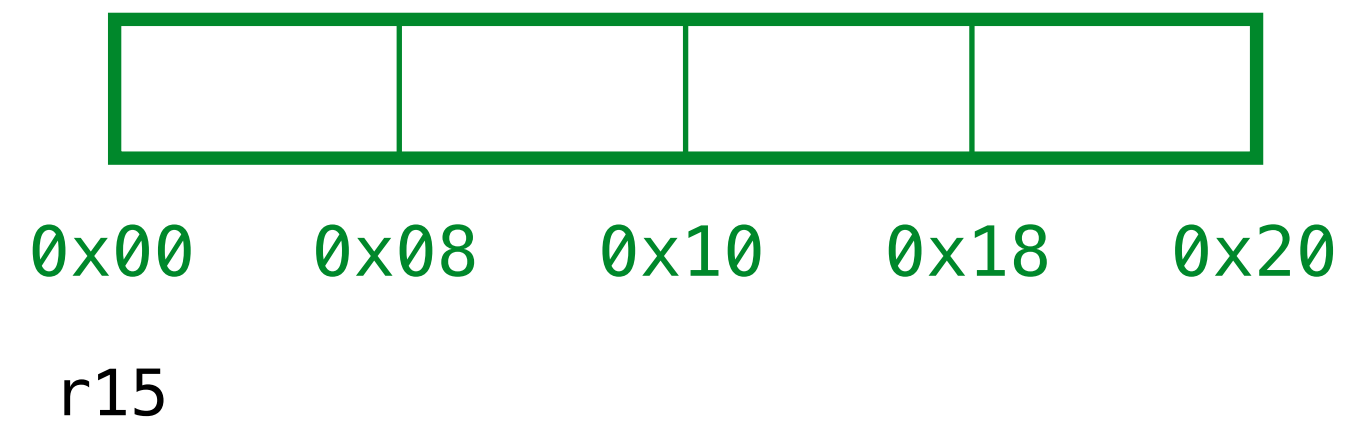
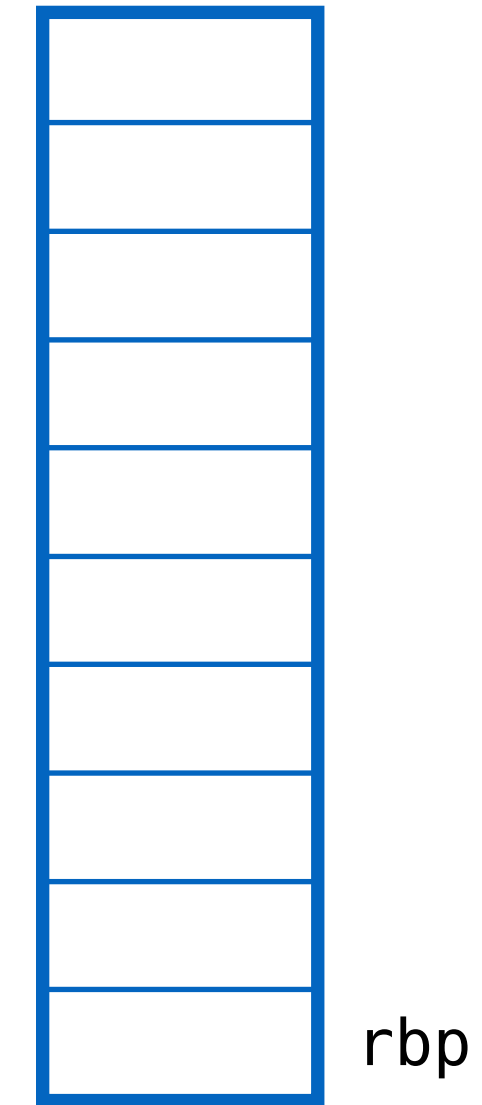


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Example 2

ex2: garbage in the middle

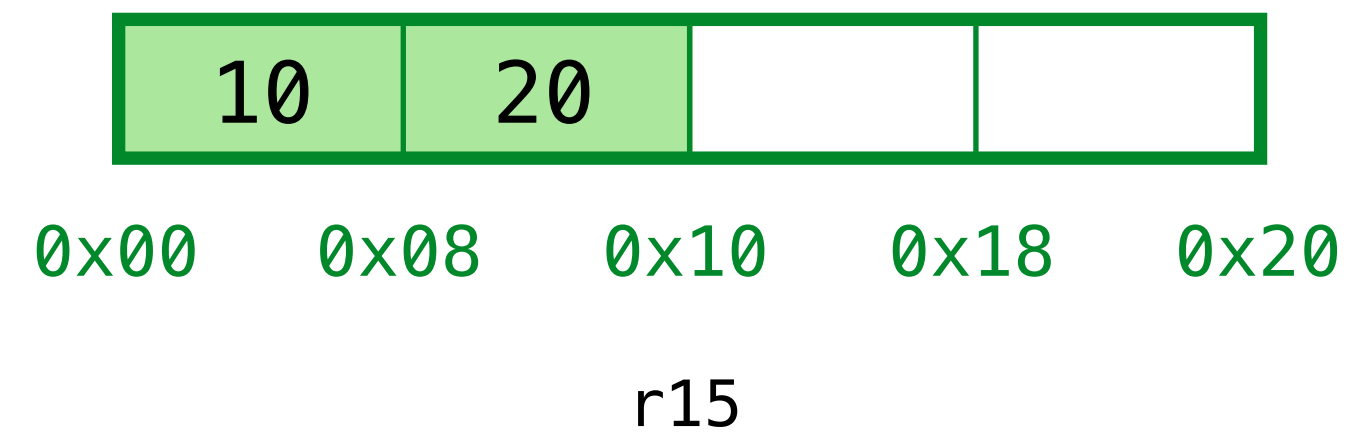
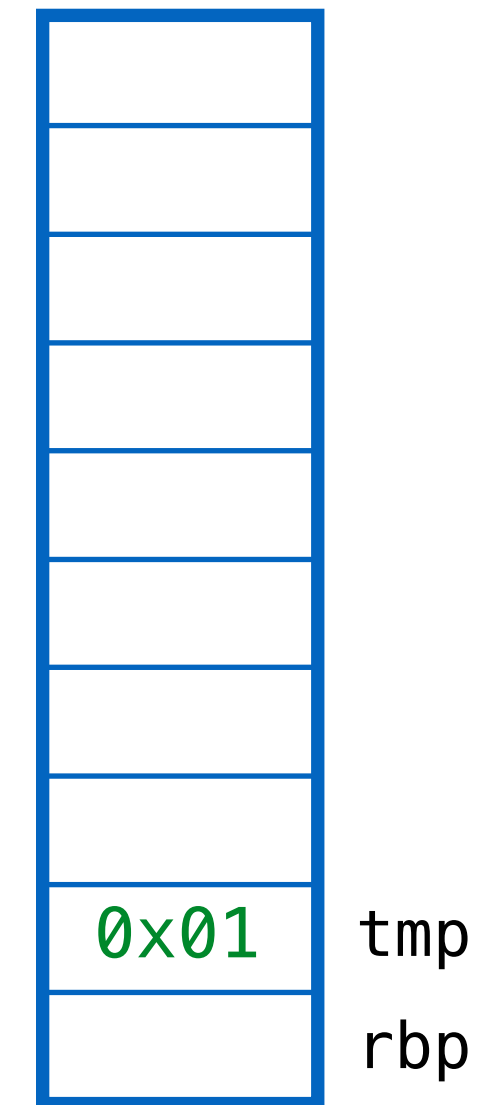
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(p0, p1)
```



Start with a 4-word heap

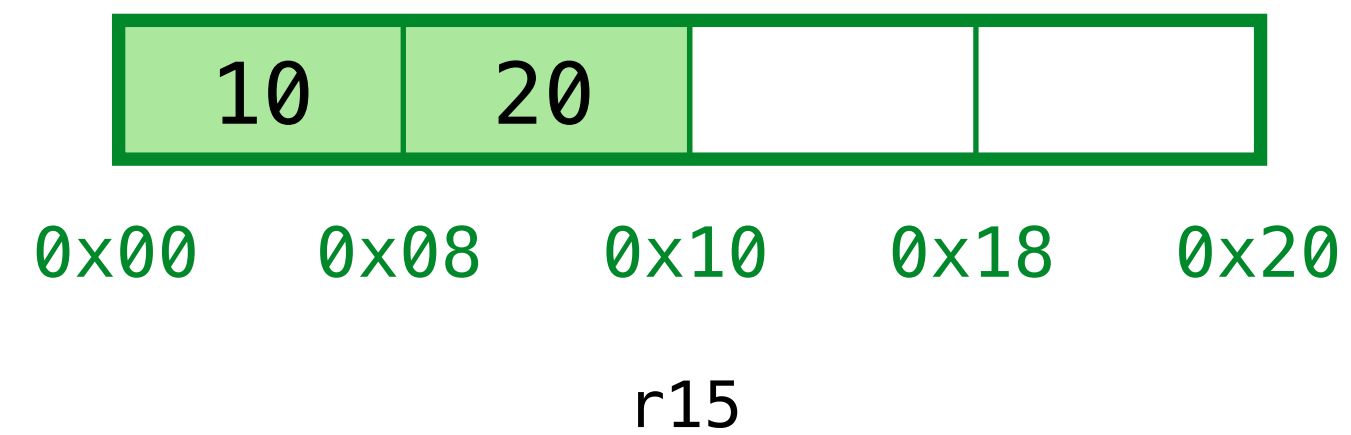
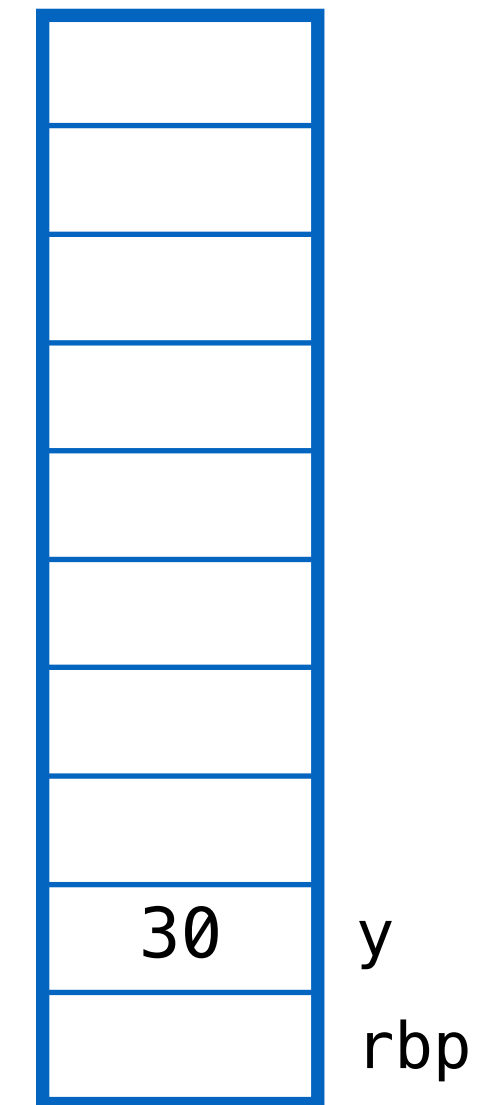
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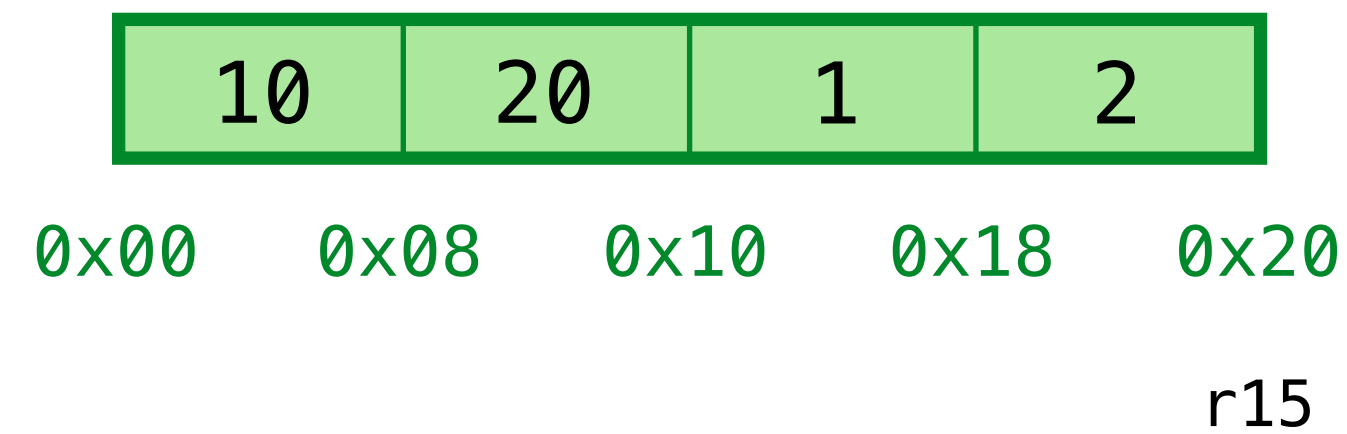
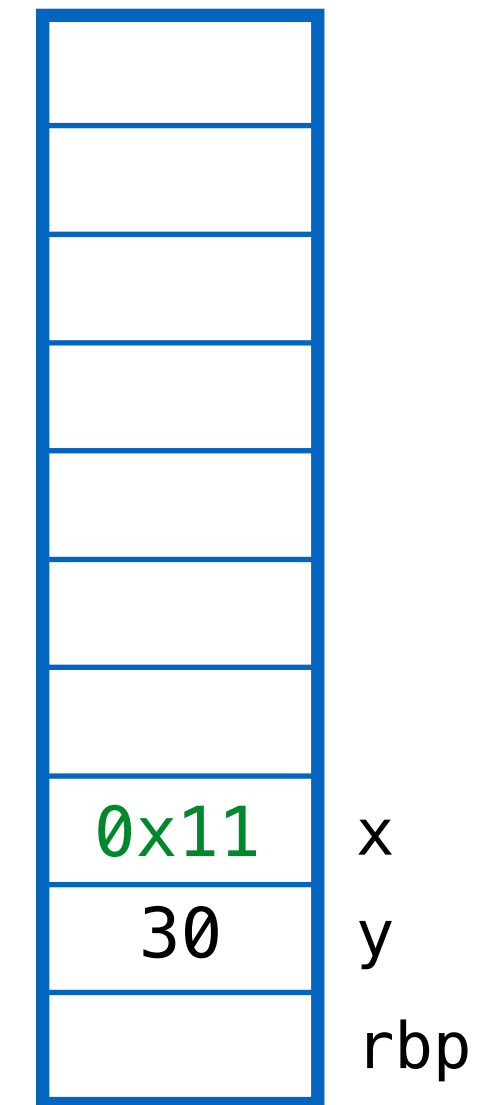
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```



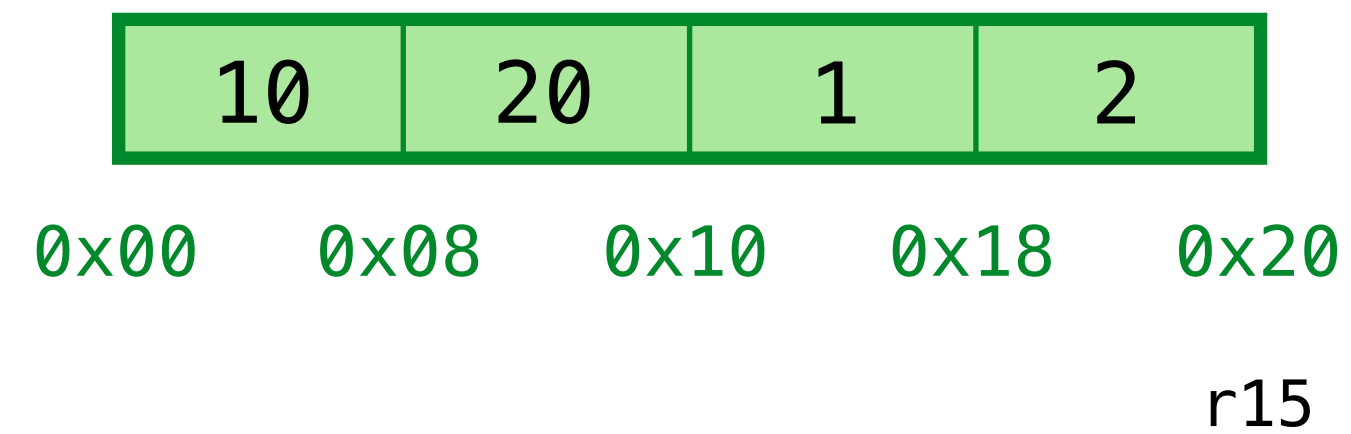
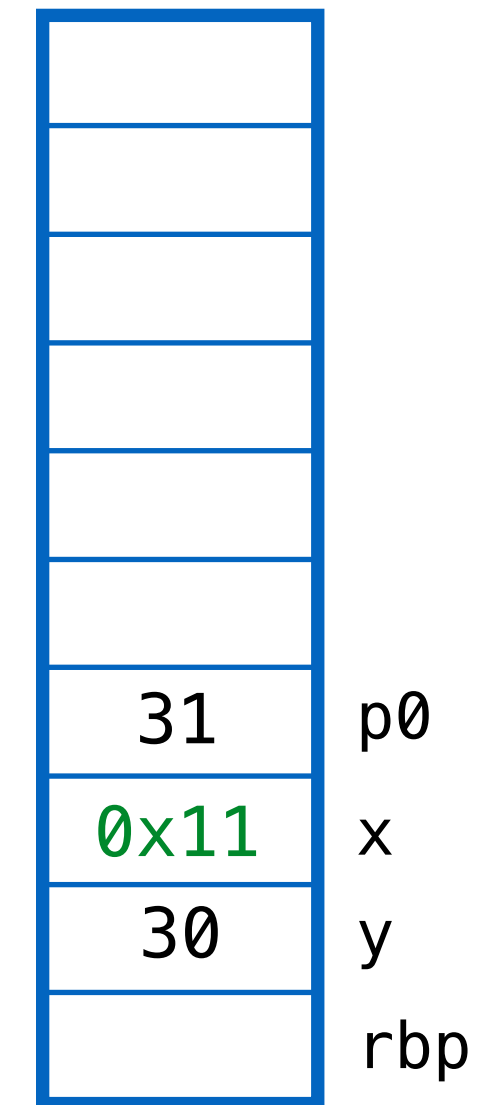
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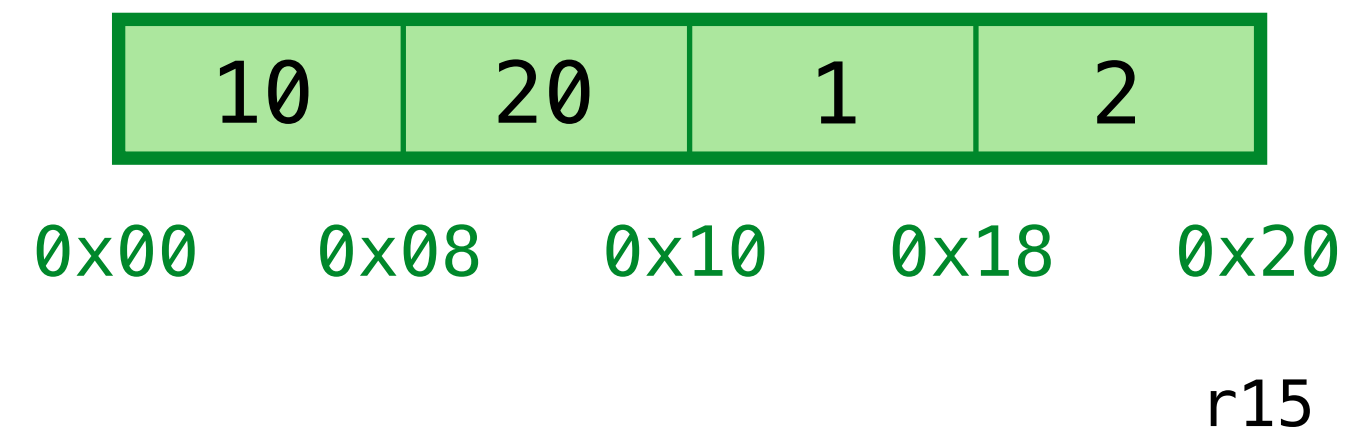
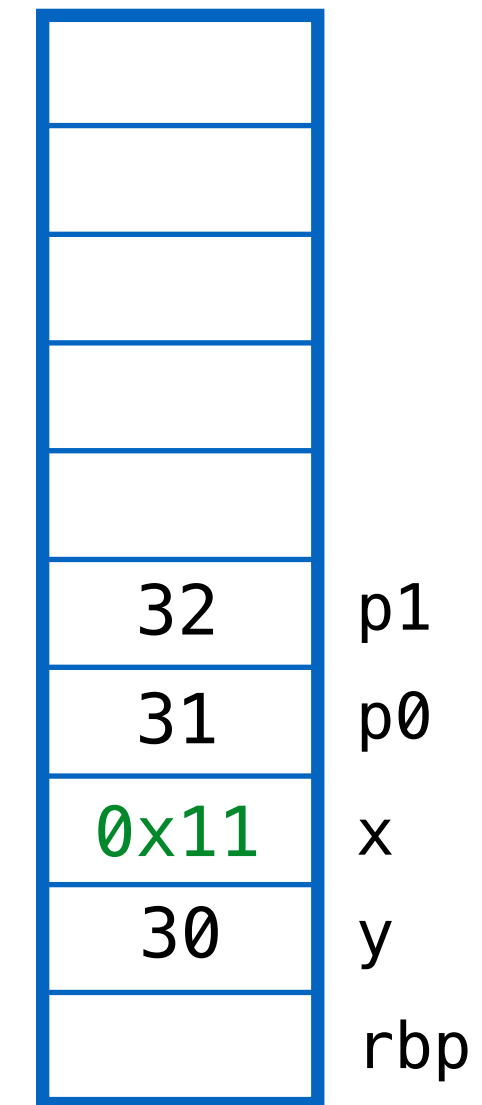
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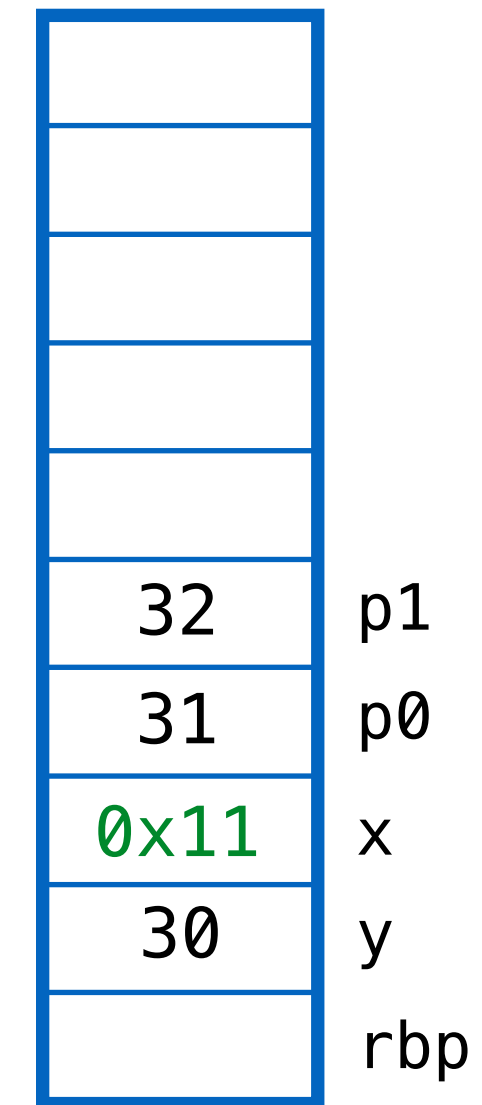
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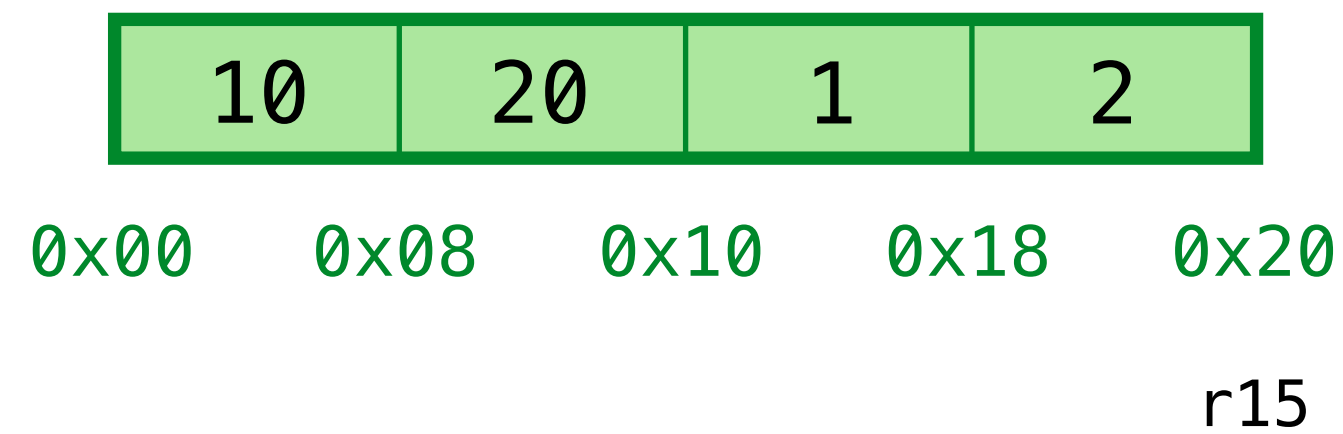


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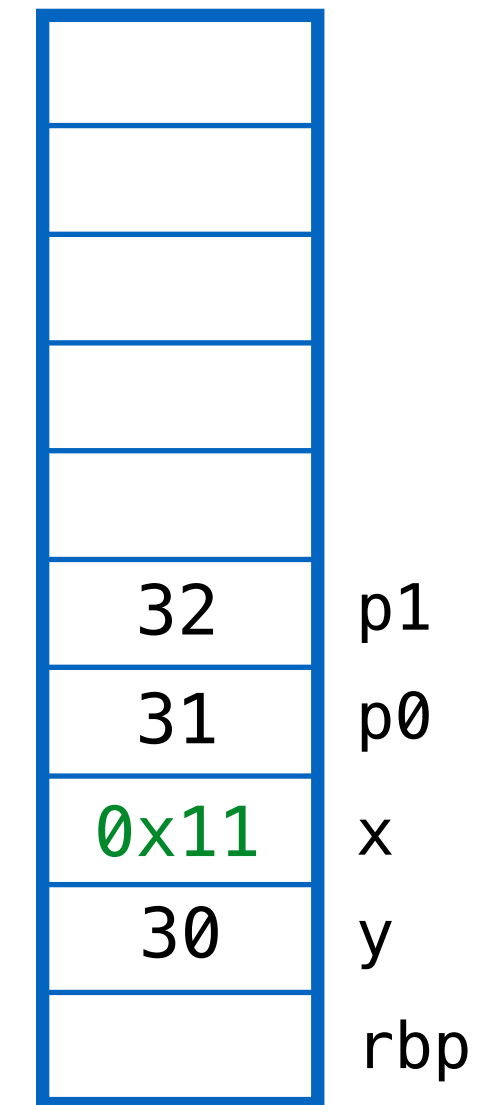


Out of memory!
Can't allocate (p0, p1)

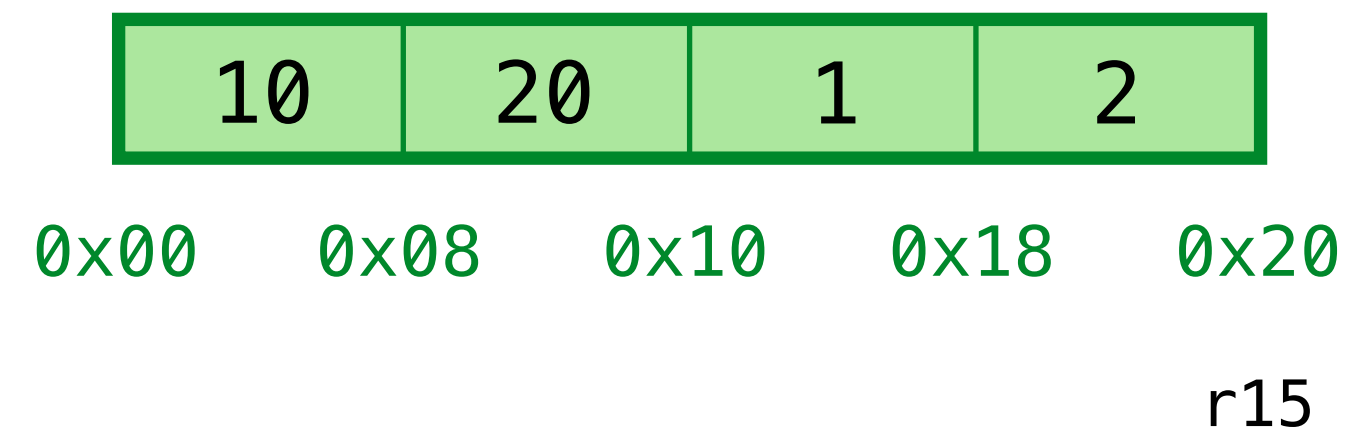


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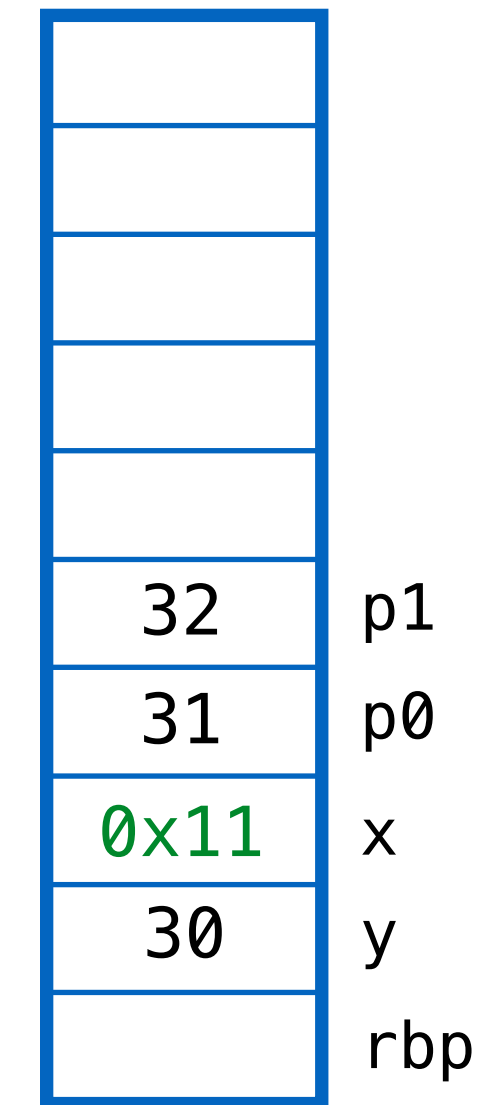


Lets reclaim & recycle garbage!

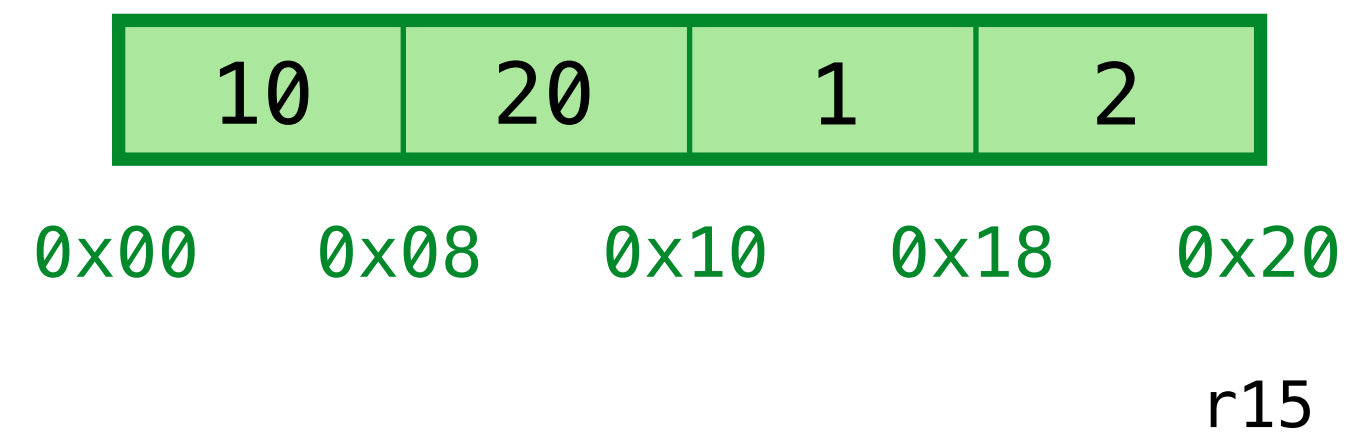


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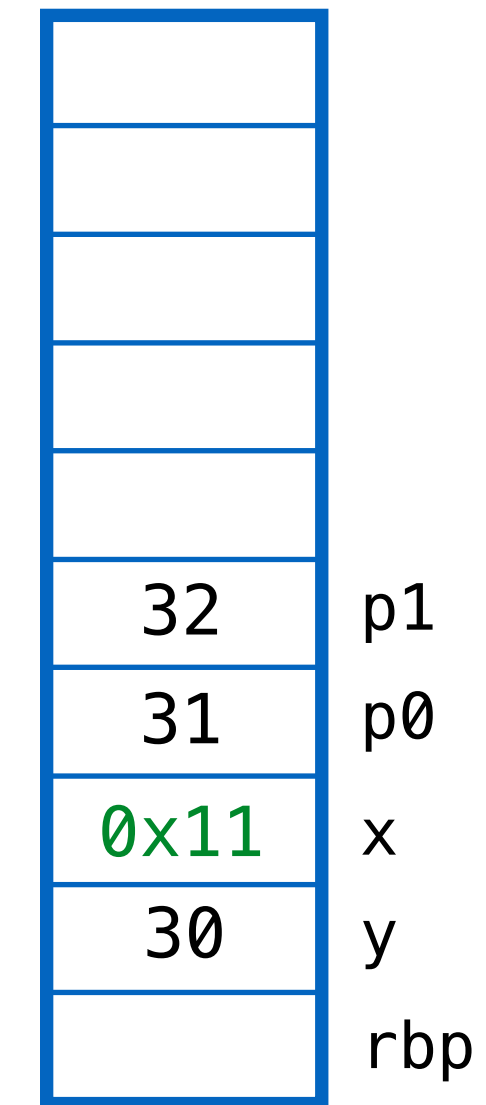


QUIZ: Which cells are garbage?

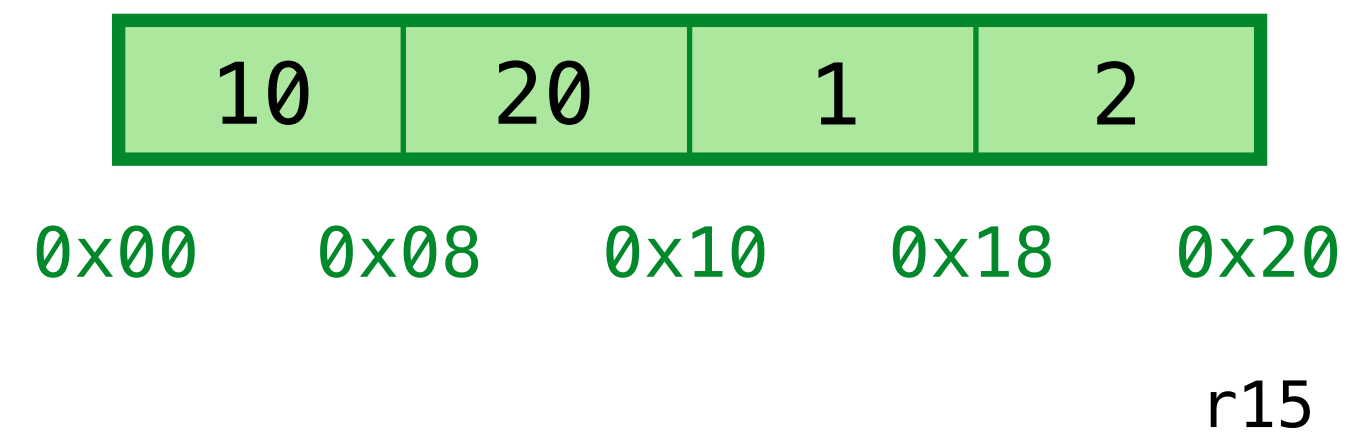
(A) 0x00, 0x08 (B) 0x08, 0x10 (C) 0x18, 0x20 (D) None (E) All

ex2: garbage in the middle

```
let y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
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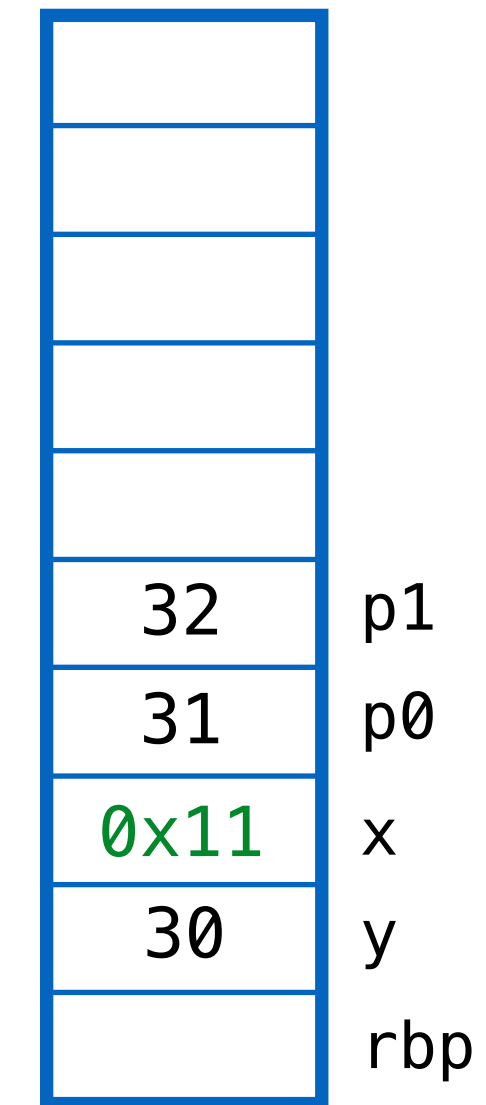


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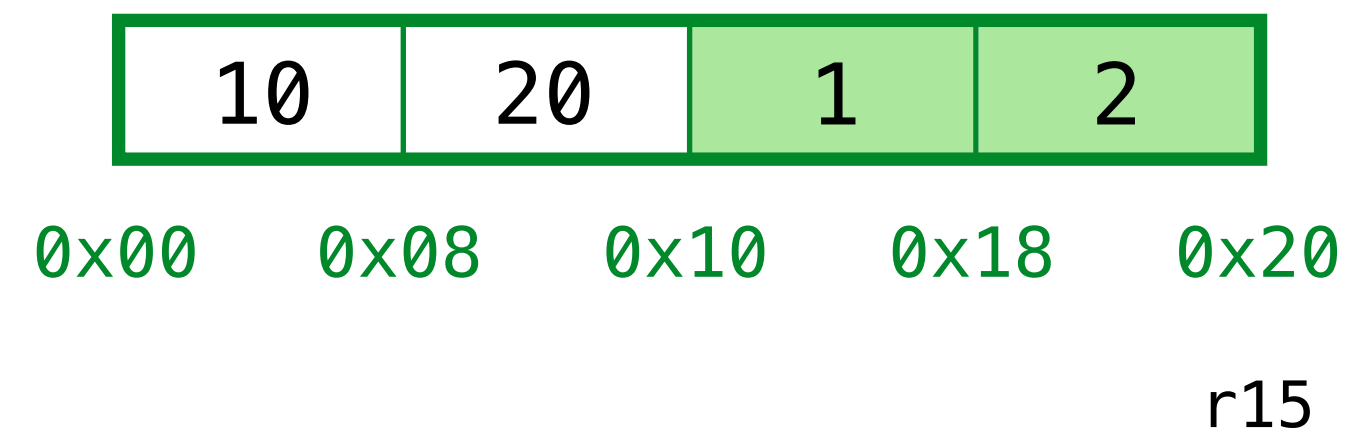
Those that are *not reachable from stack*

ex2: garbage in the middle

```
let y = let tmp = (10, 20)
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, x = (1, 2)
, p0 = x[0] + y
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```



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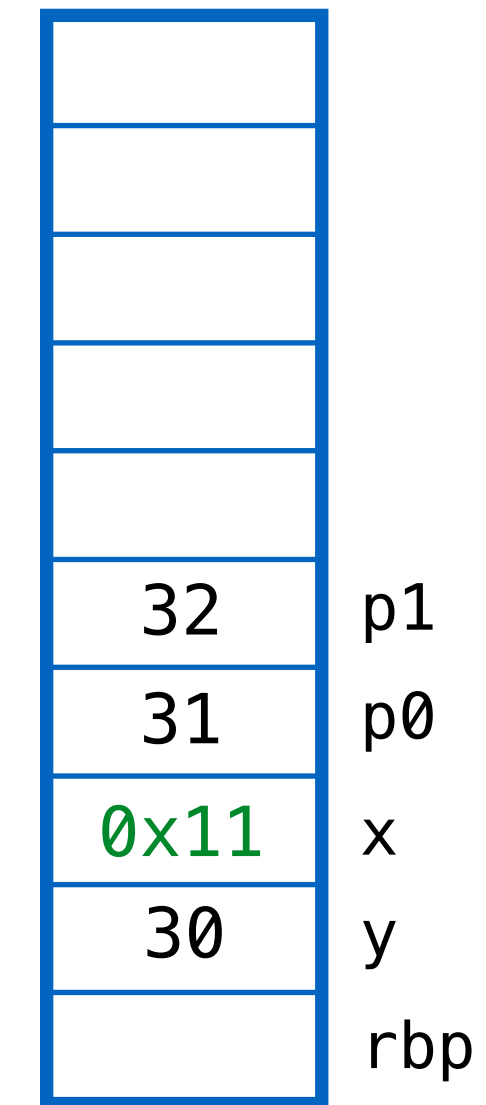


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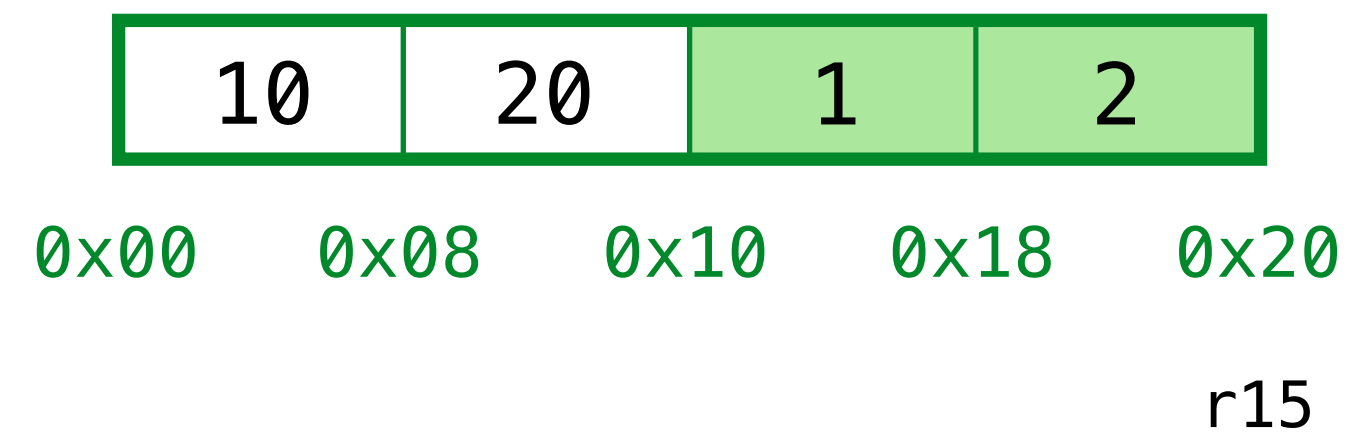
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in (p0, p1)
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Lets reclaim & recycle garbage!

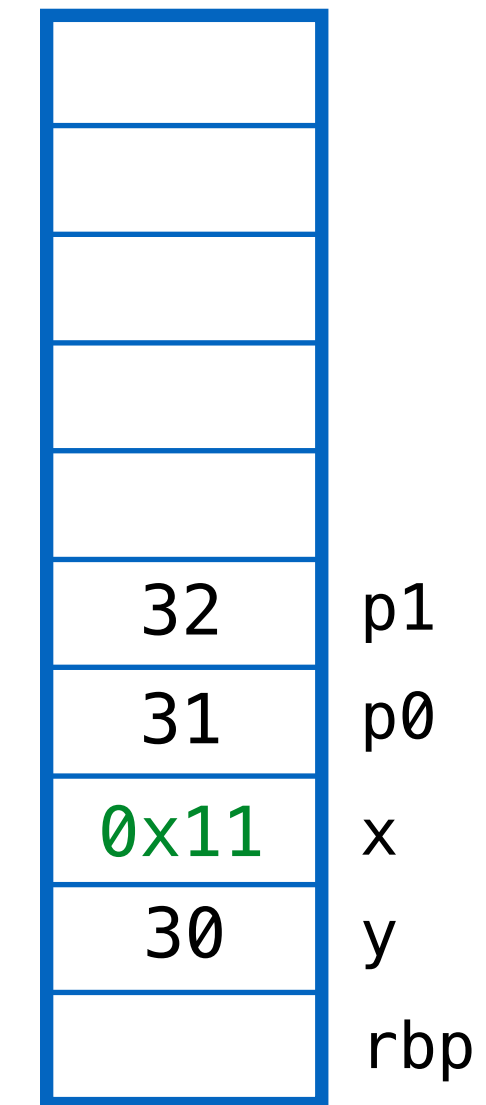


Q: How to reclaim space?

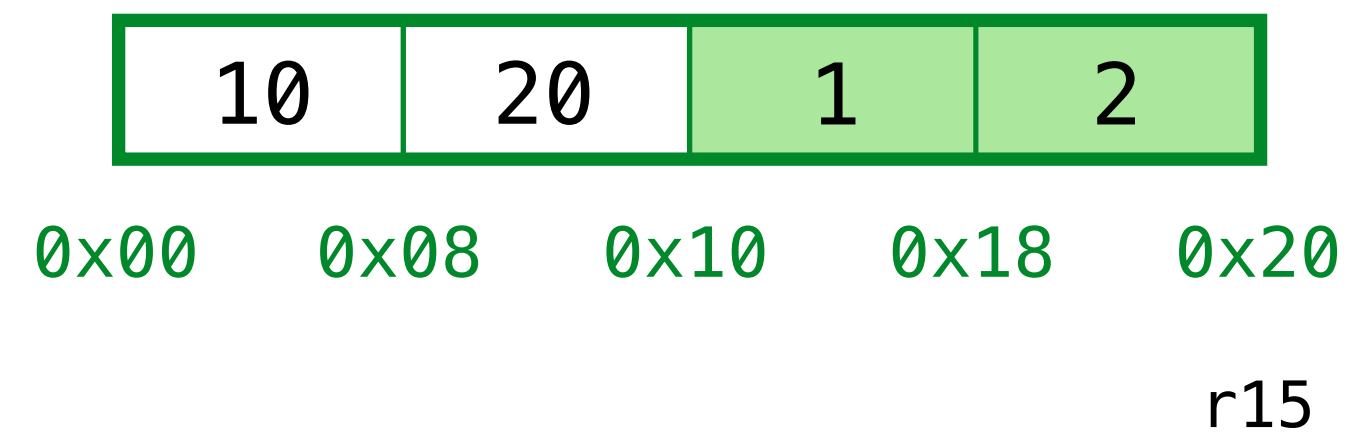
Why is it not enough to rewind r15?

ex2: garbage in the middle

```
let y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
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, p0 = x[0] + y
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Lets reclaim & recycle garbage!

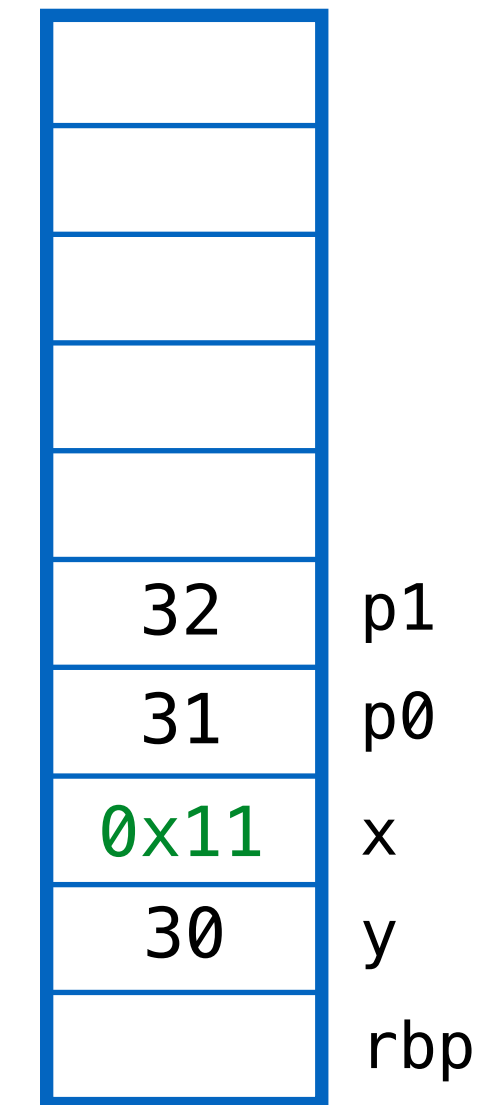


Why is it not enough to rewind r15?

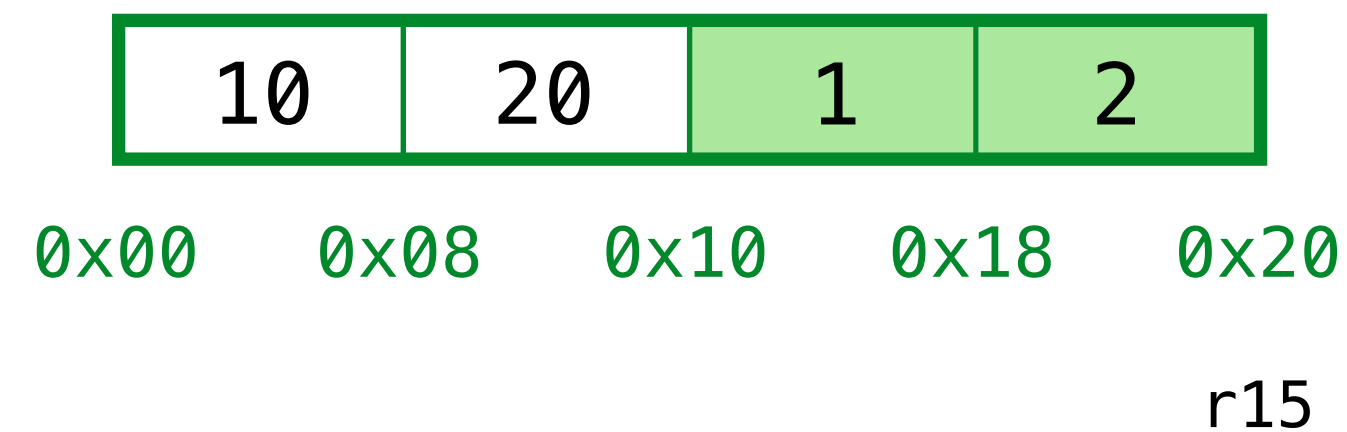
Want free space to be *contiguous* (i.e. go to end of heap)

ex2: garbage in the middle

```
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        in tmp[0] + tmp[1]
, x = (1, 2)
, p0 = x[0] + y
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Lets reclaim & recycle garbage!

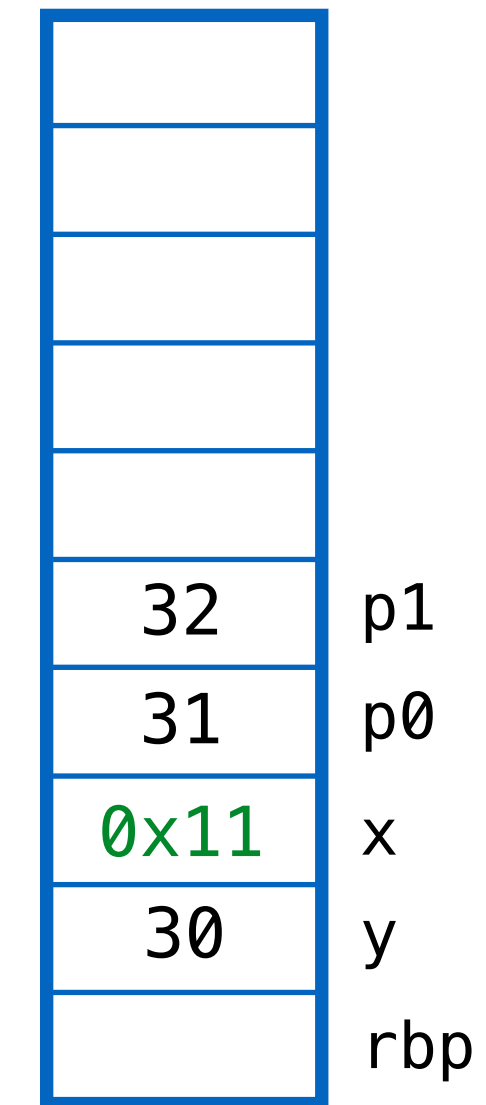


Solution: Compaction

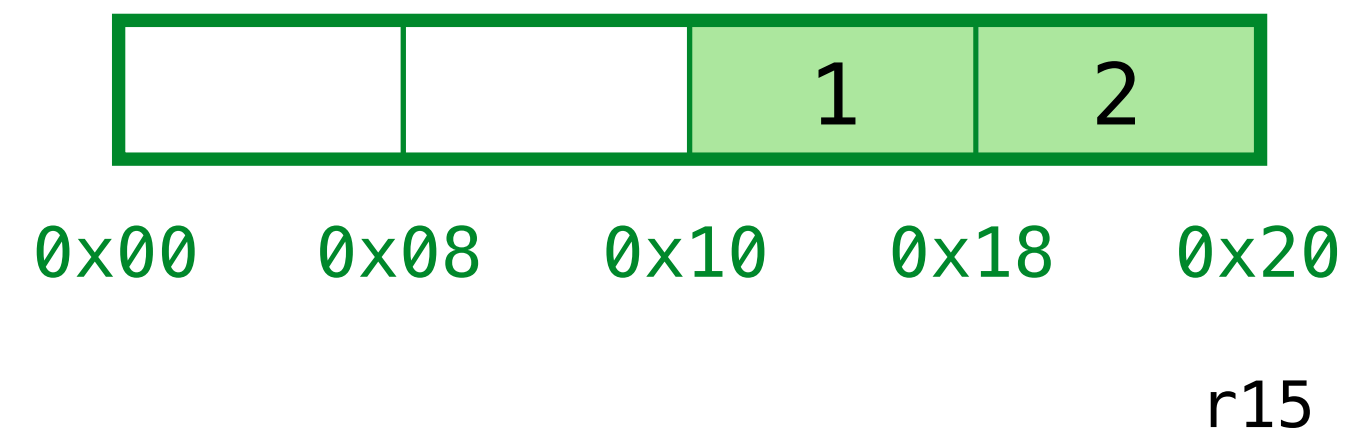
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y = let tmp = (10, 20)
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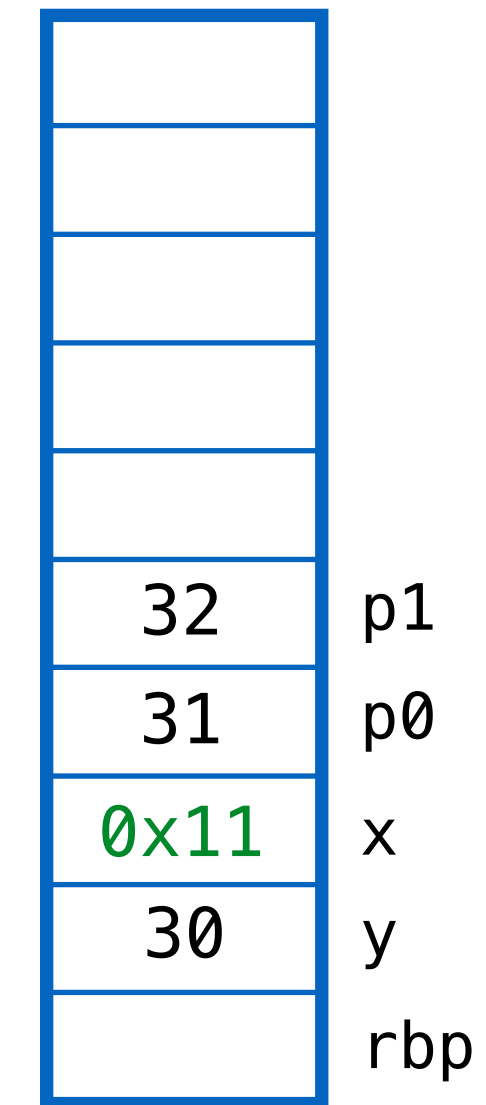


Solution: Compaction

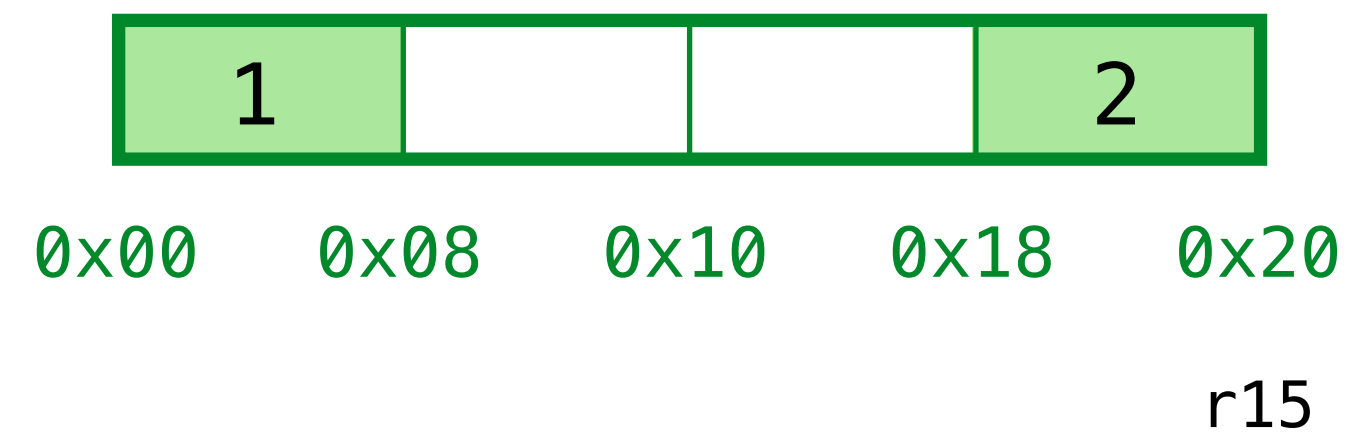
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in (p0, p1)
```



Lets reclaim & recycle garbage!

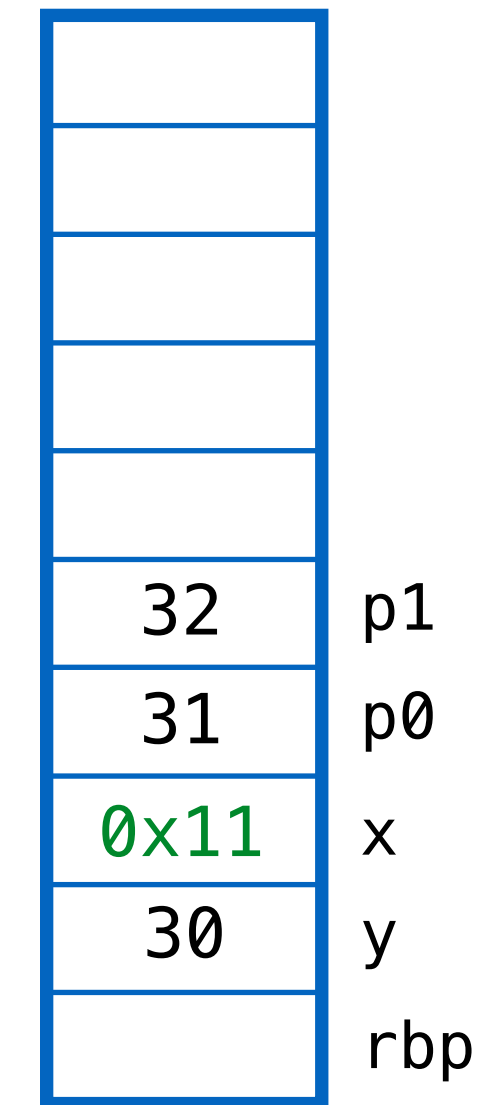


Solution: Compaction

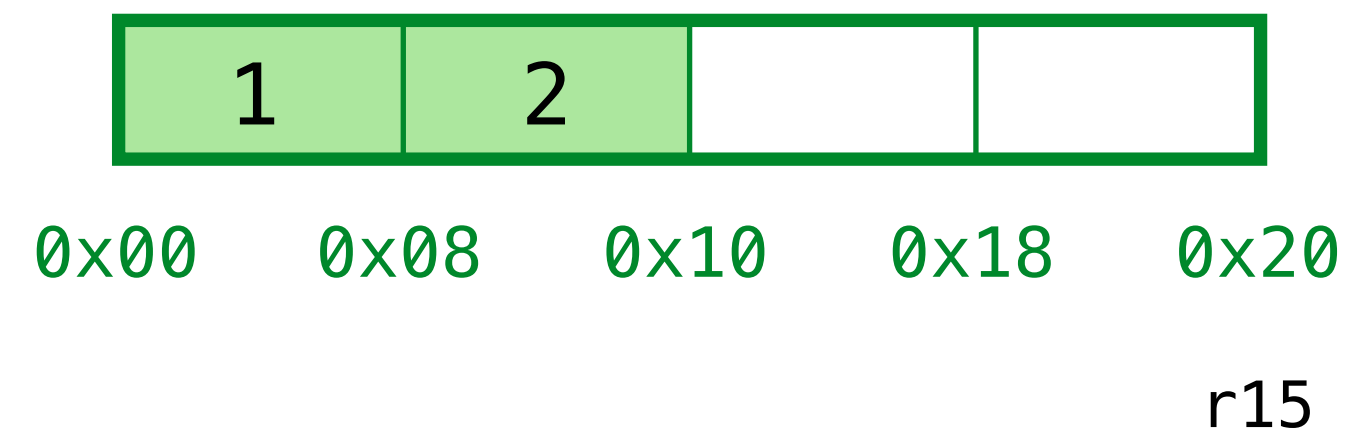
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in (p0, p1)
```



Lets reclaim & recycle garbage!

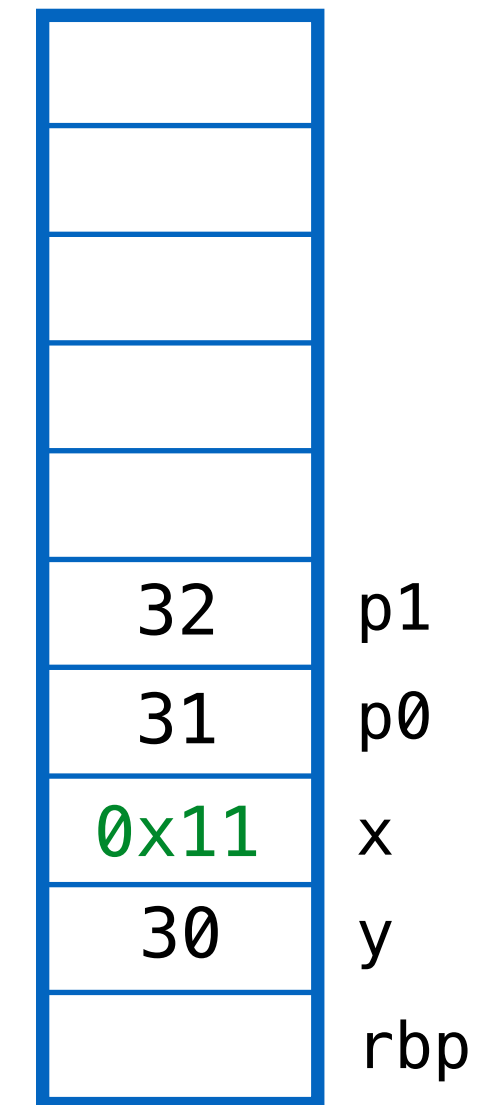


Solution: Compaction

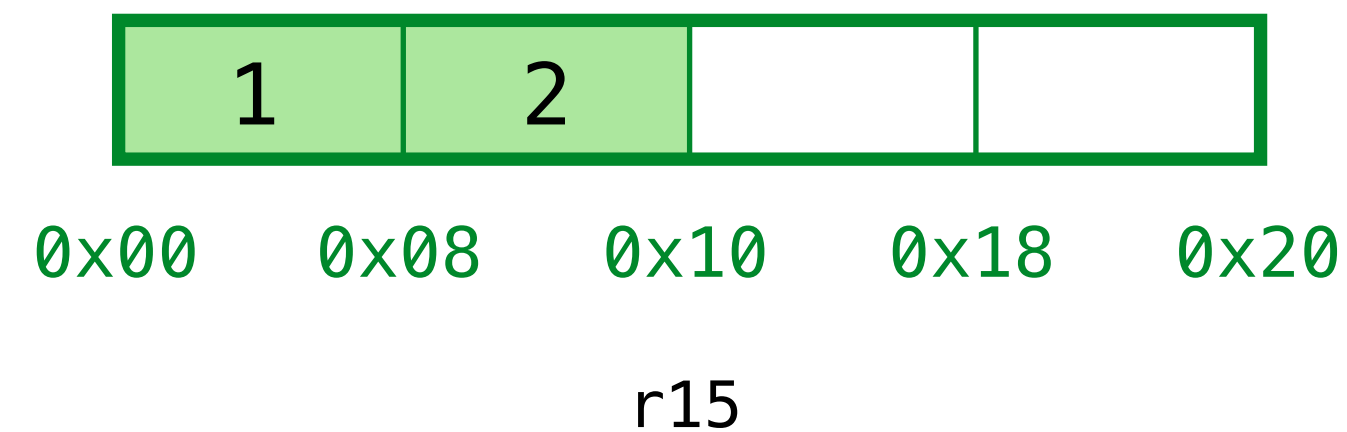
Copy “live” cells into “garbage” ...

ex2: garbage in the middle

```
let y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in (p0, p1)
```



Lets reclaim & recycle garbage!

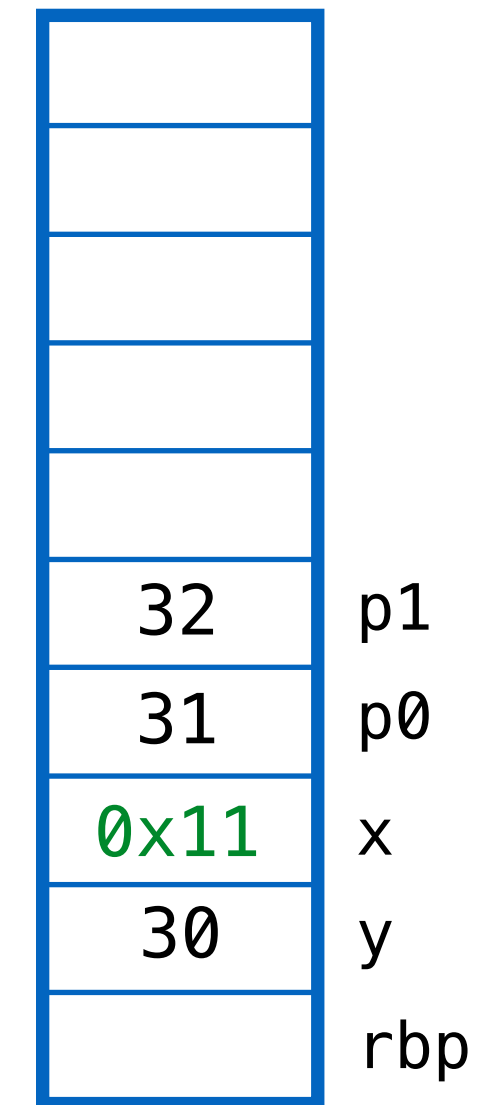


Solution: Compaction

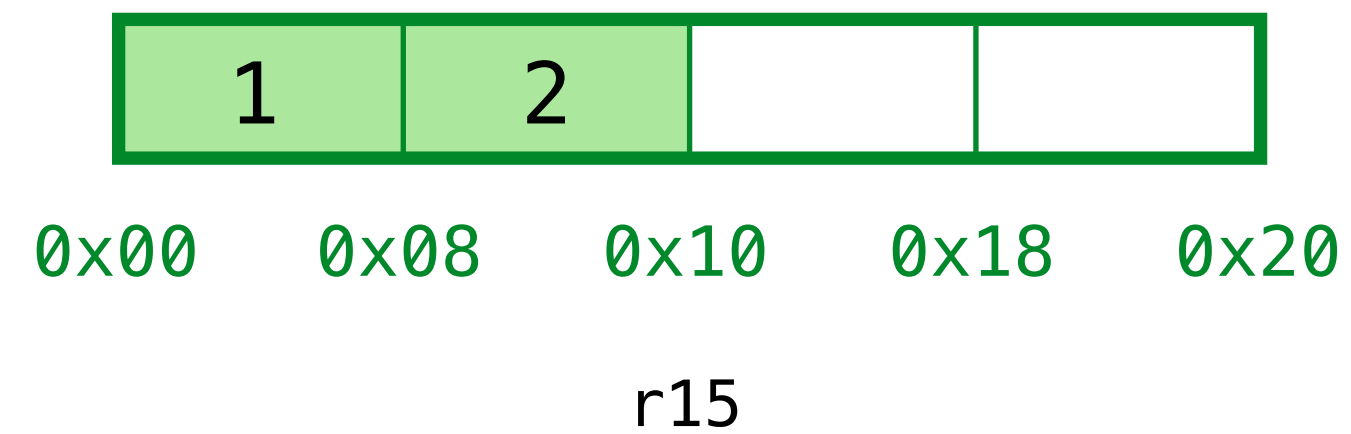
Copy “live” cells into “garbage” ... *and then* ... rewind r15!

ex2: garbage in the middle

```
let y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in (p0, p1)
```

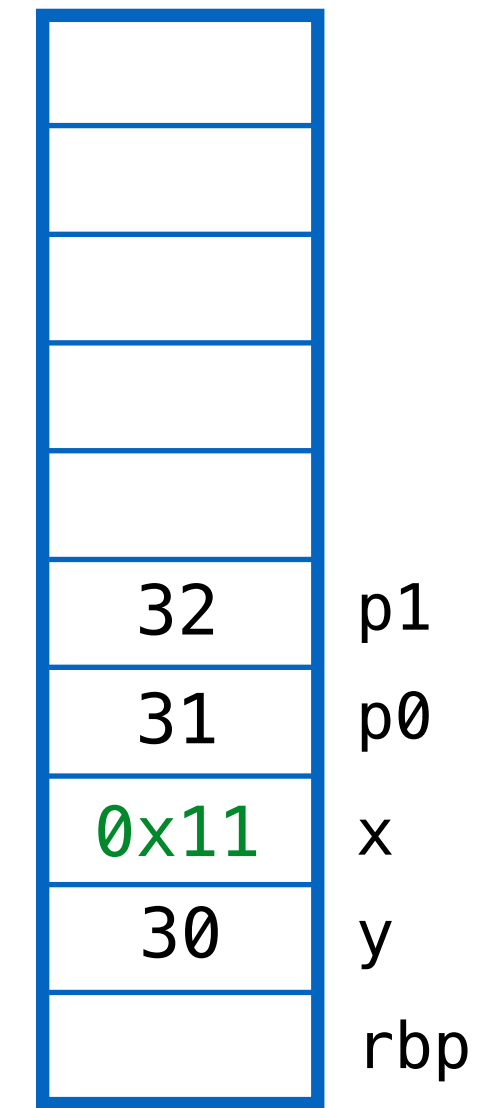


Yay! Have space for (p0, p1)

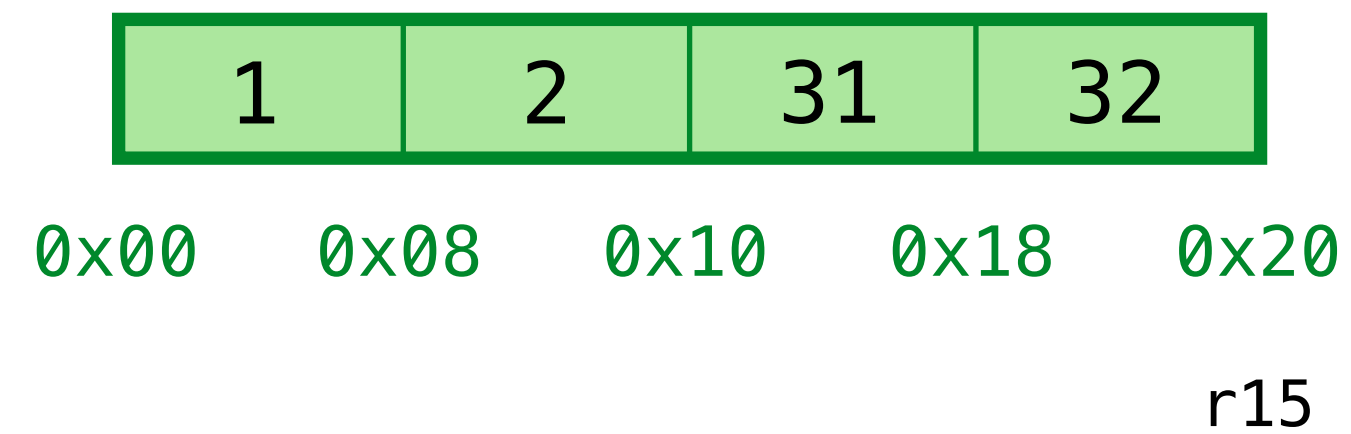


ex2: garbage in the middle

```
let y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```

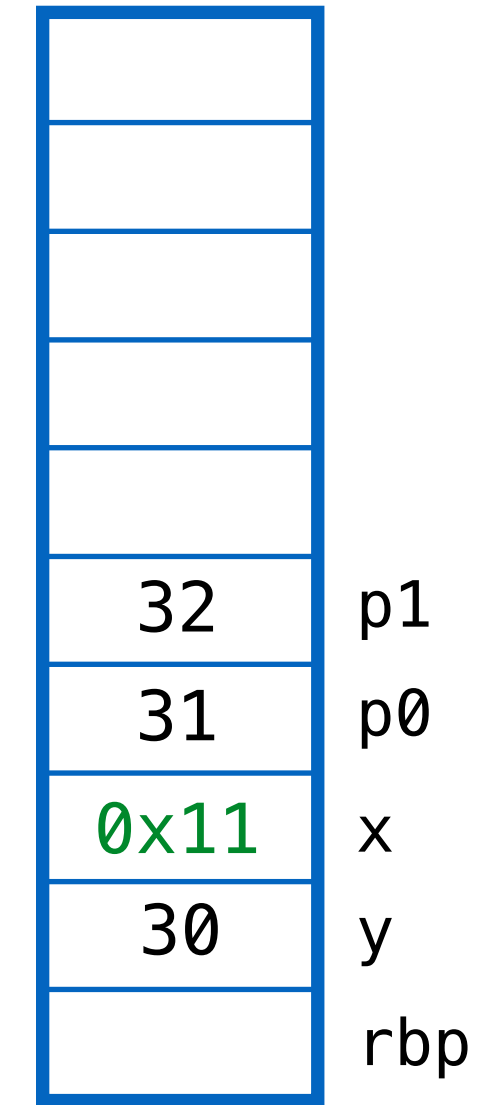


Yay! Have space for (p0, p1)

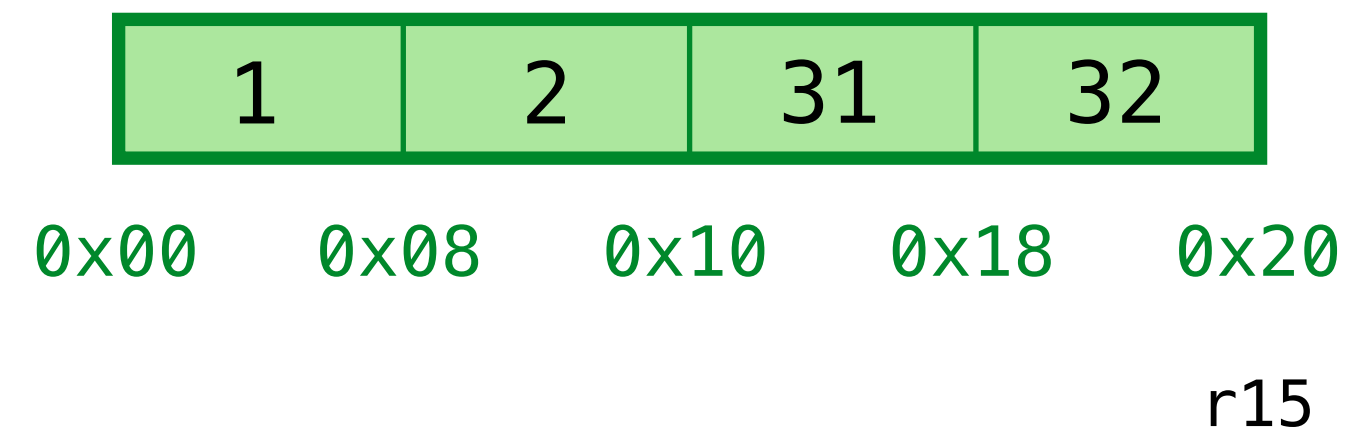


ex2: garbage in the middle

```
let y = let tmp = (10, 20)
        in tmp[0] + tmp[1]
, x = (1, 2)
, p0 = x[0] + y
, p1 = x[1] + y
in
(p0, p1)
```



Result (rax) = 0x09

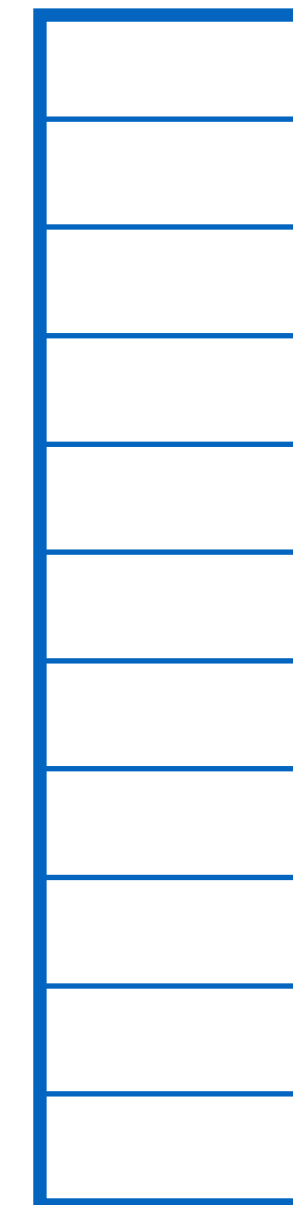


Garter / GC

Example 3

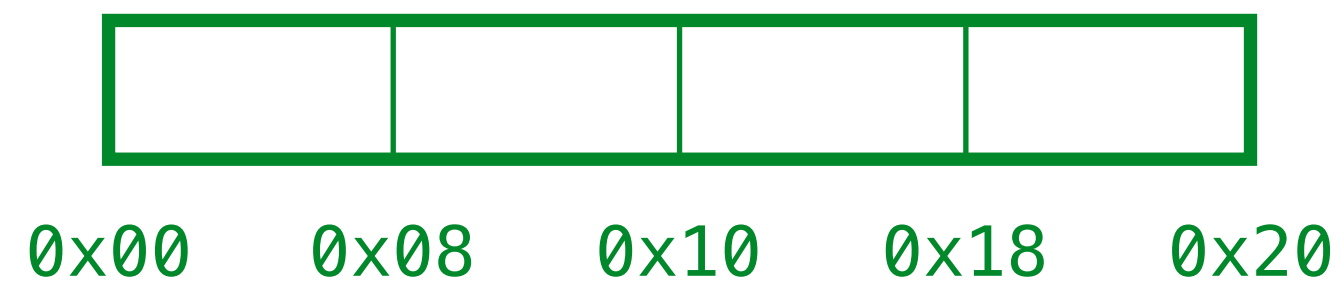
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + y + z
```



rsp
|
|
|
|
|
|
|
|
|
|
rbp

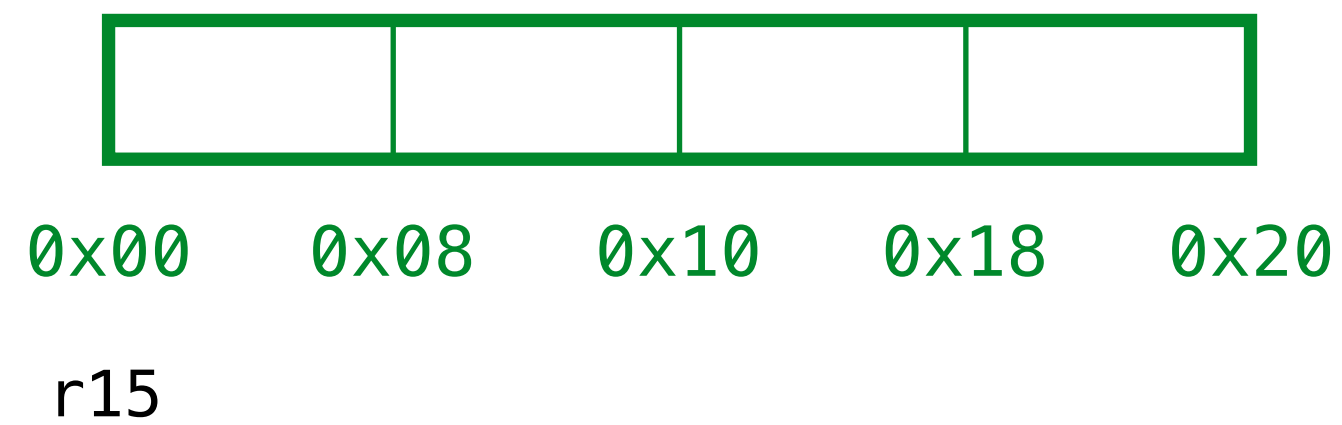
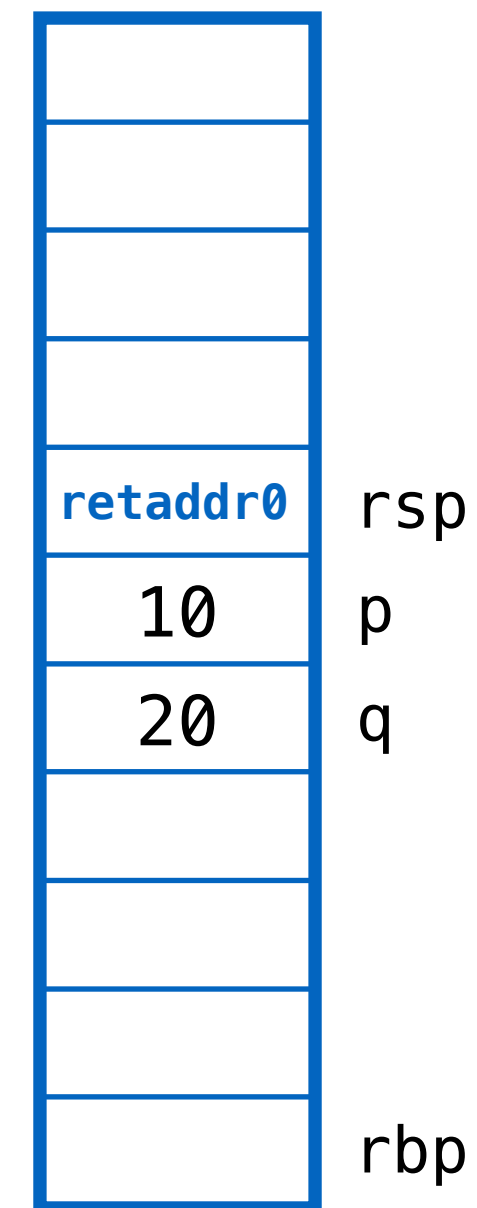
3 local vars x, y, z



r15

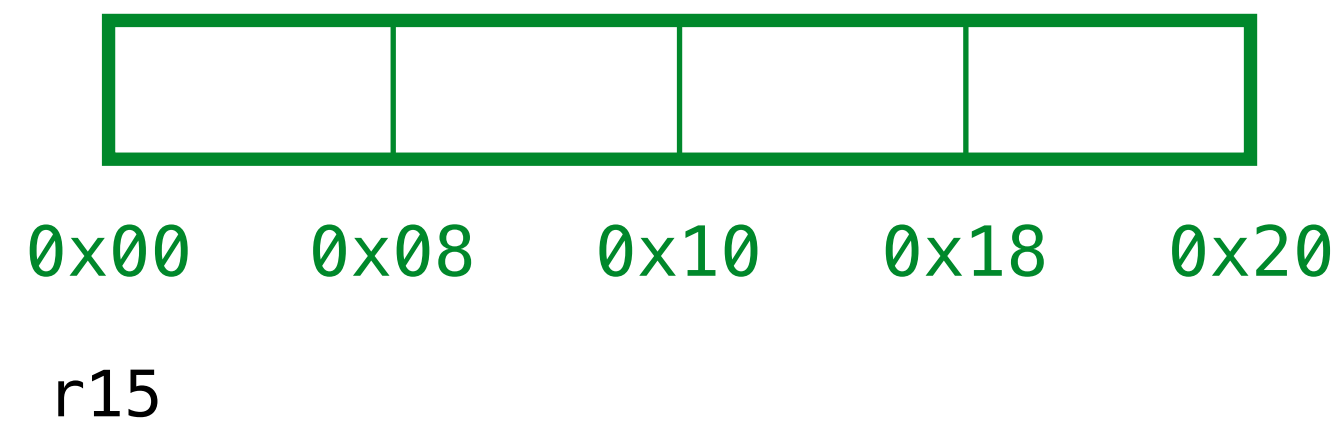
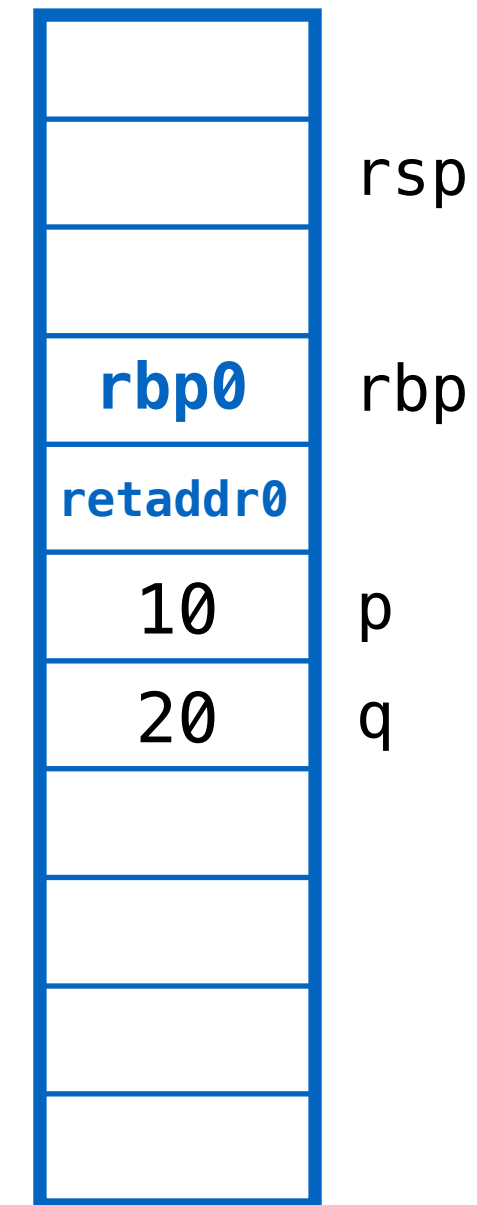
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20) ←  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + y + z
```



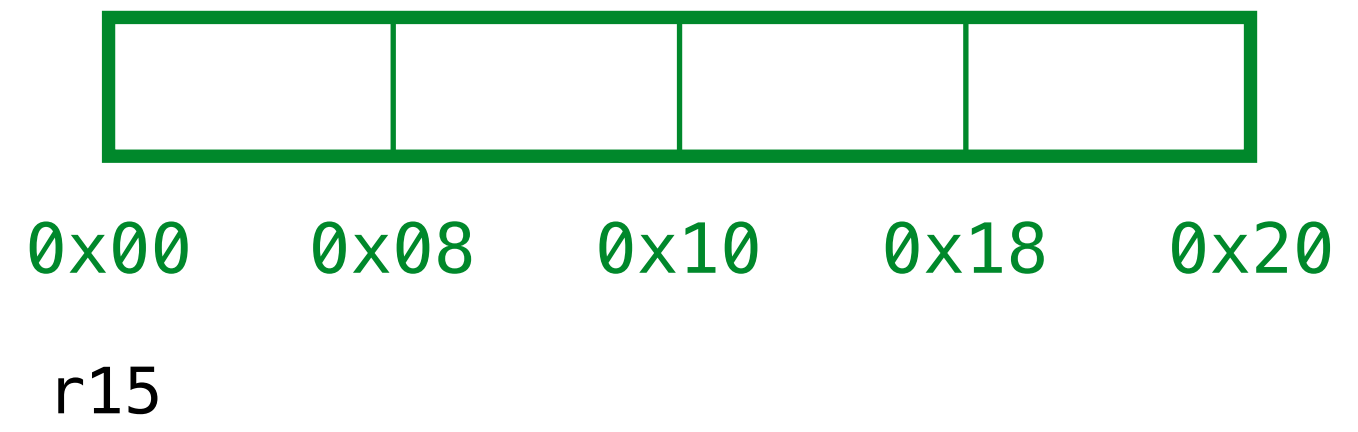
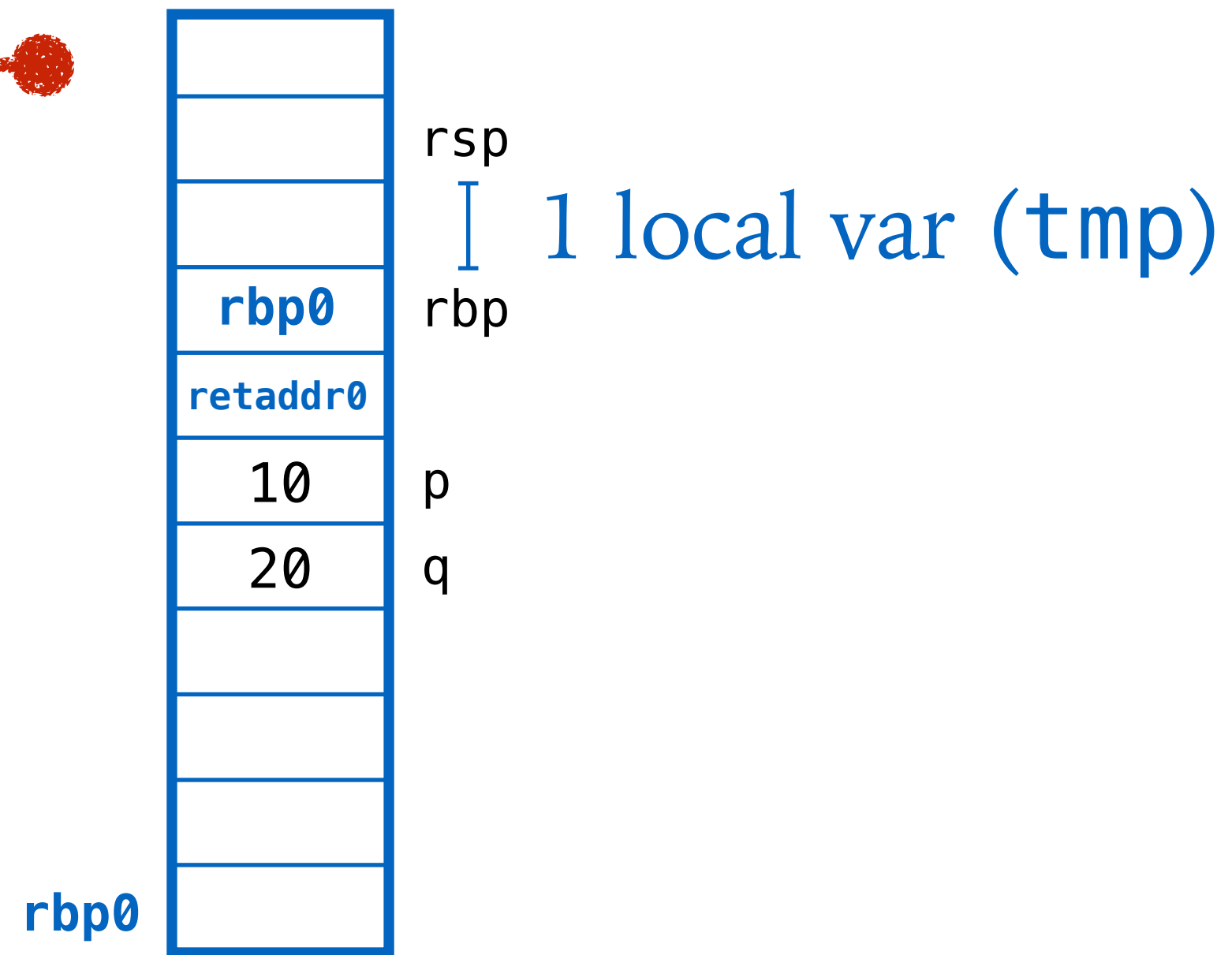
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + y + z
```



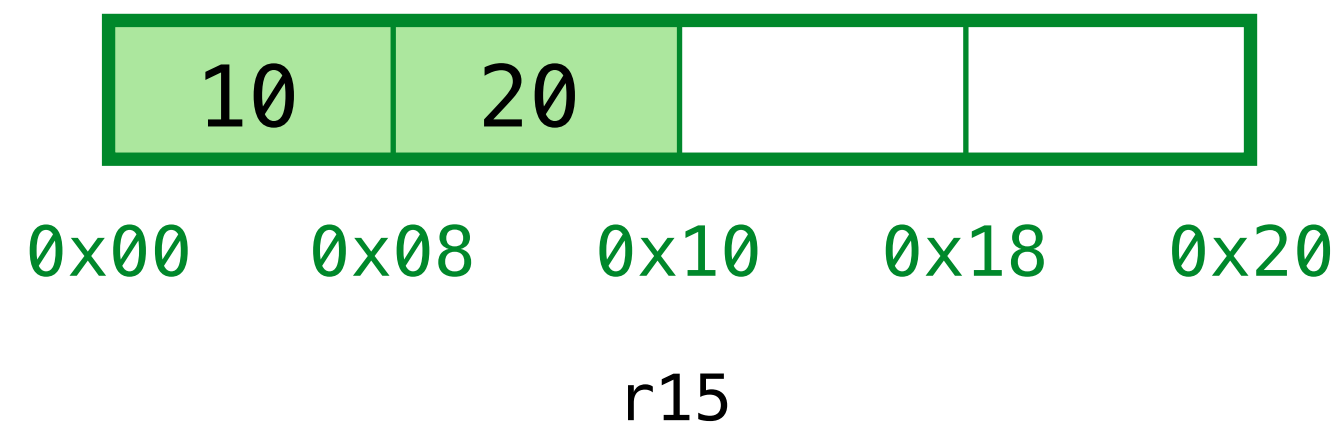
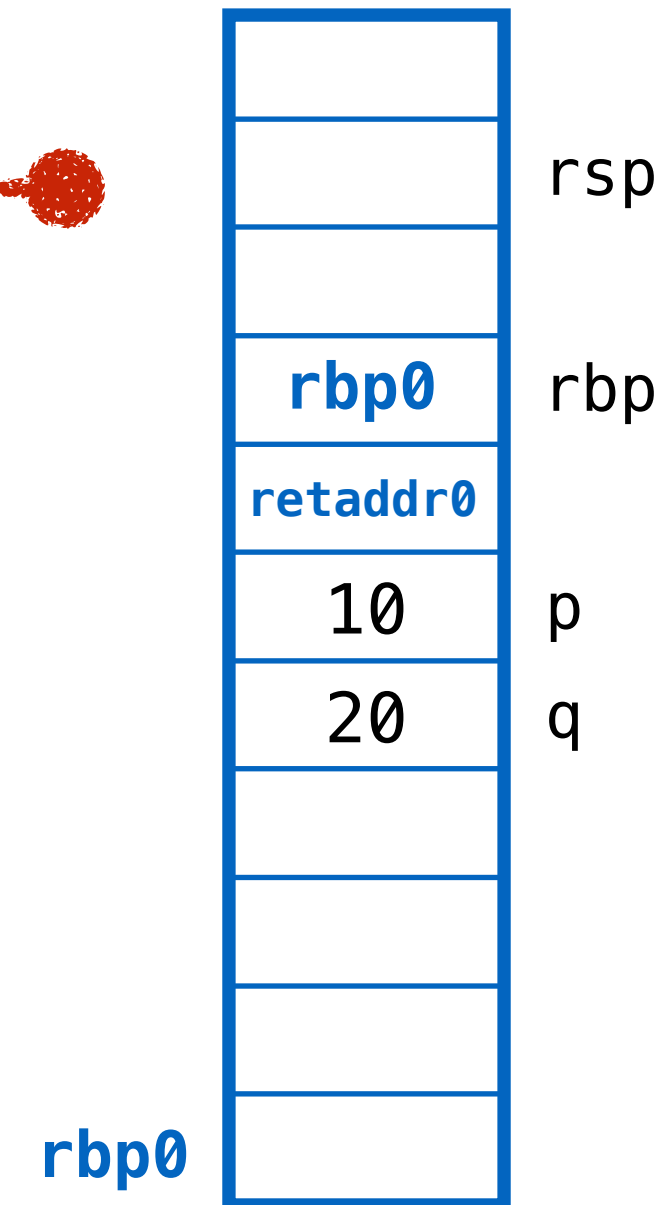
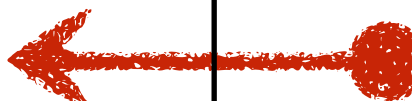
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + y + z
```



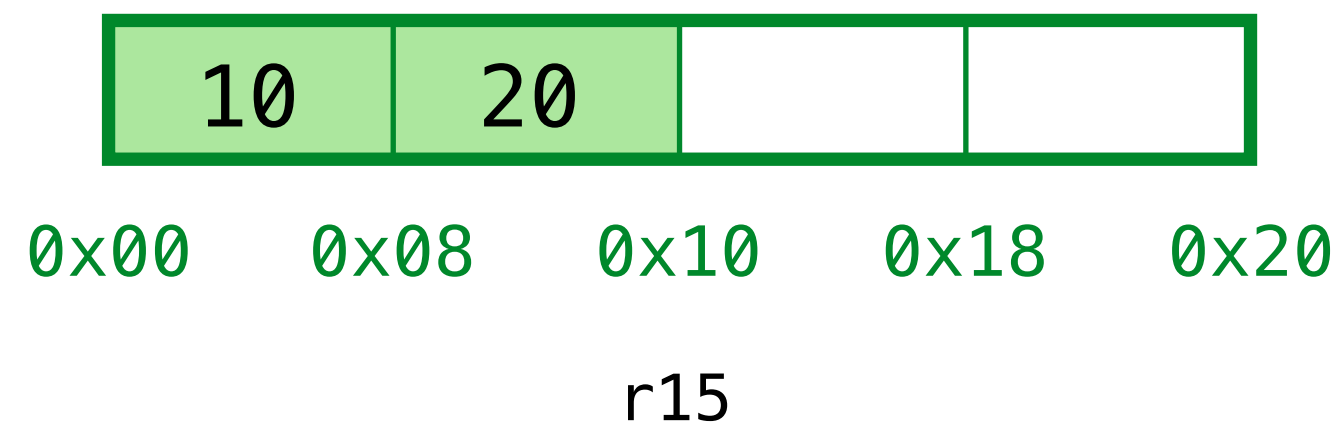
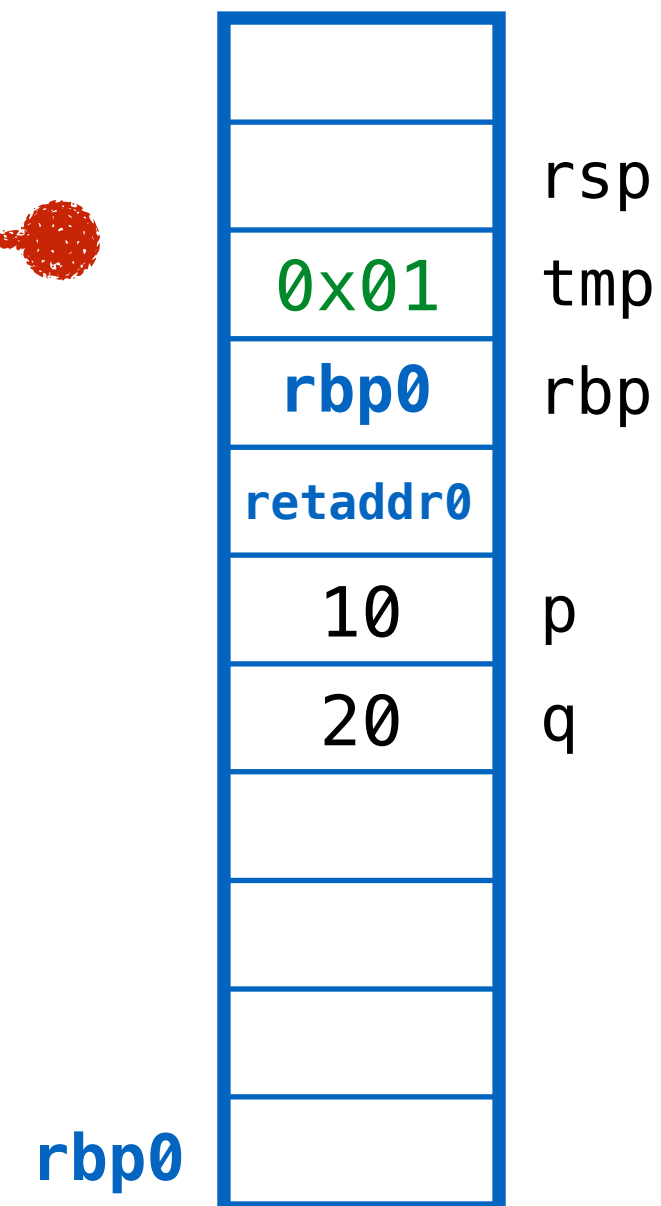
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + y + z
```



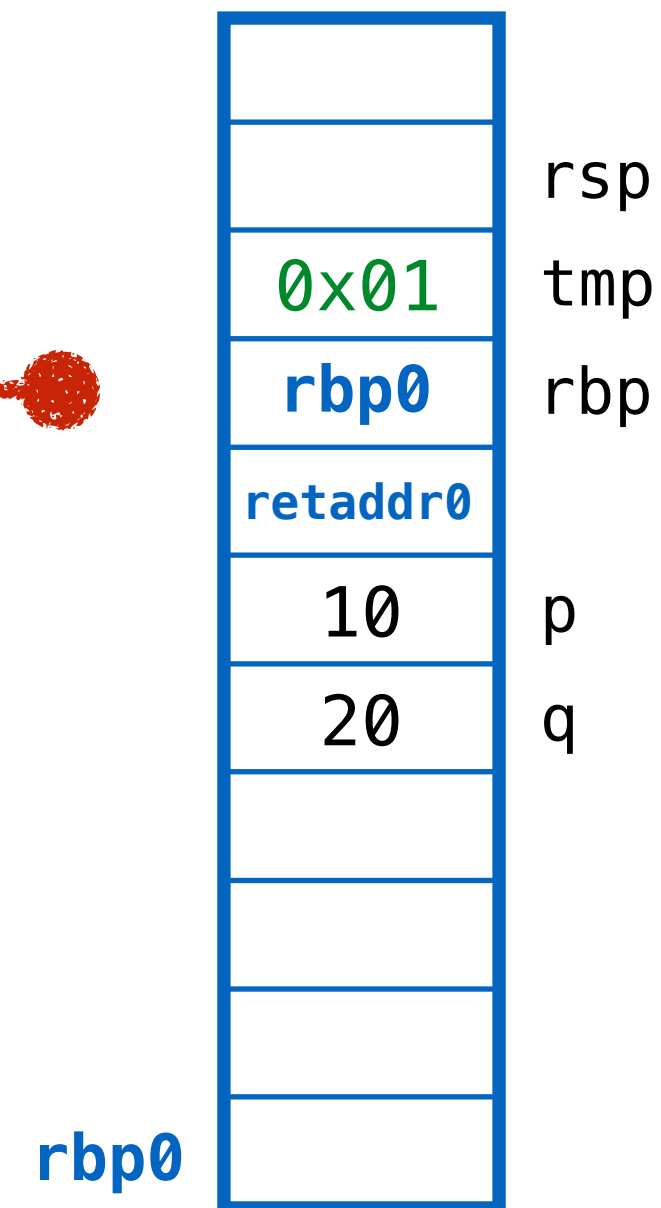
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + y + z
```

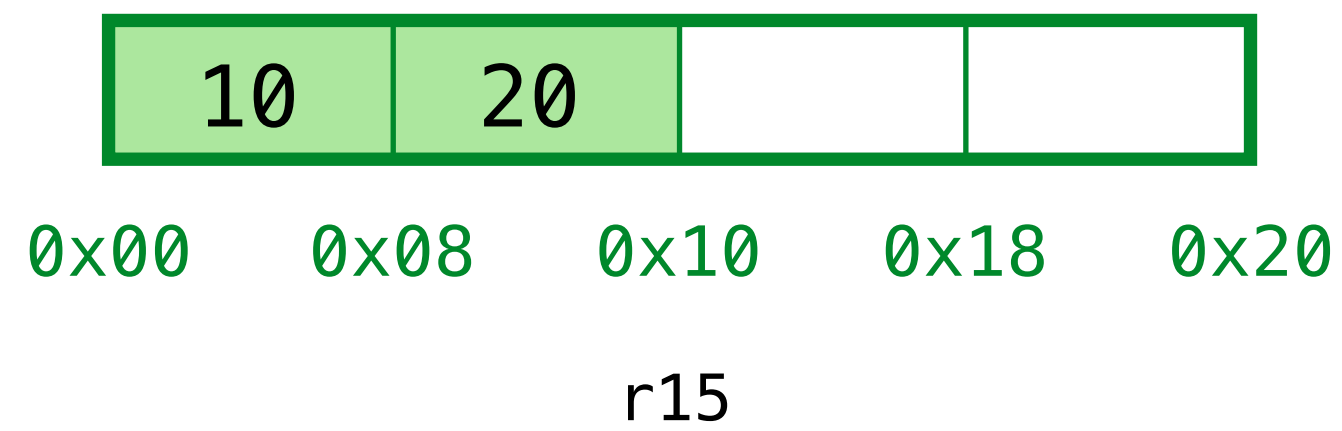


ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + y + z
```

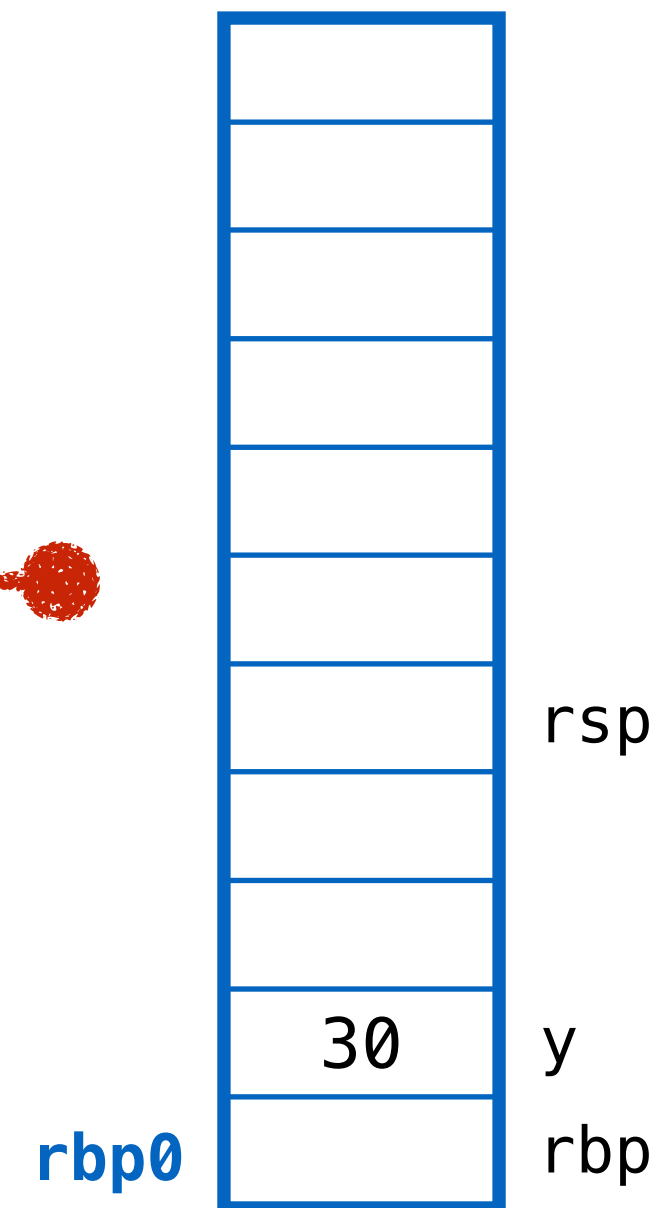


Return (rax) = 30

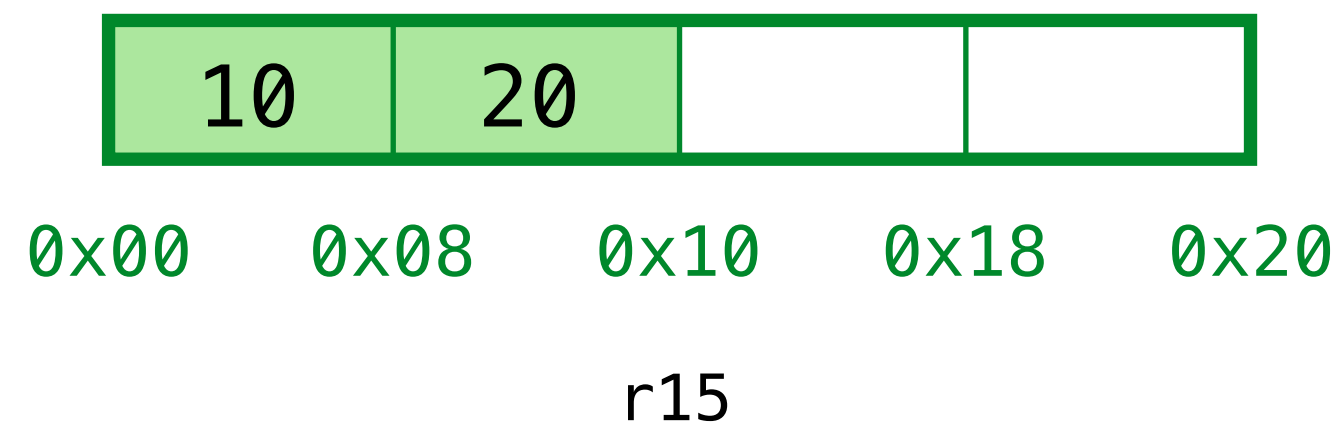


ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
  , x = (y, y + 1)  
  , z = foo(100, 200)  
in  
  x[0] + z
```

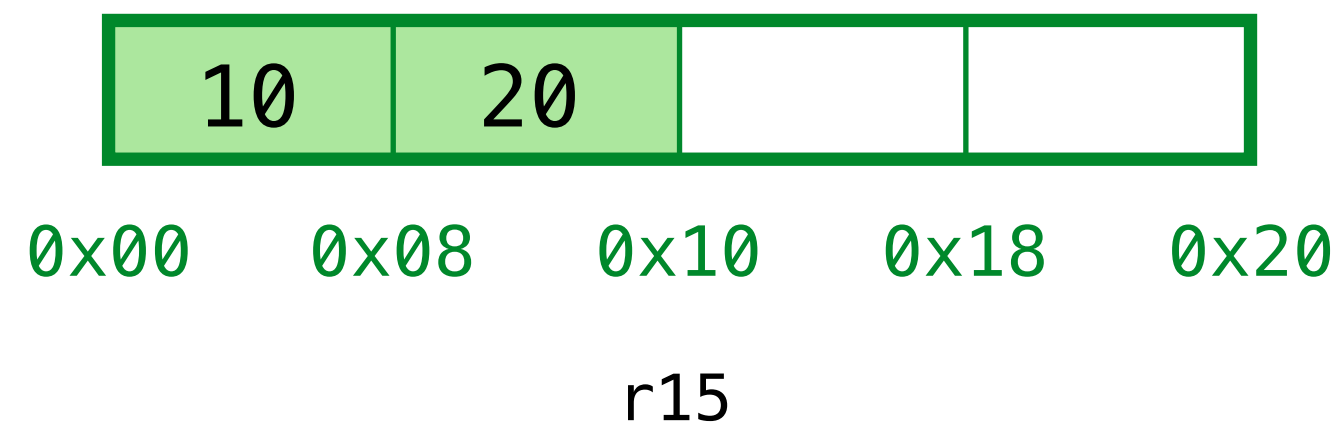
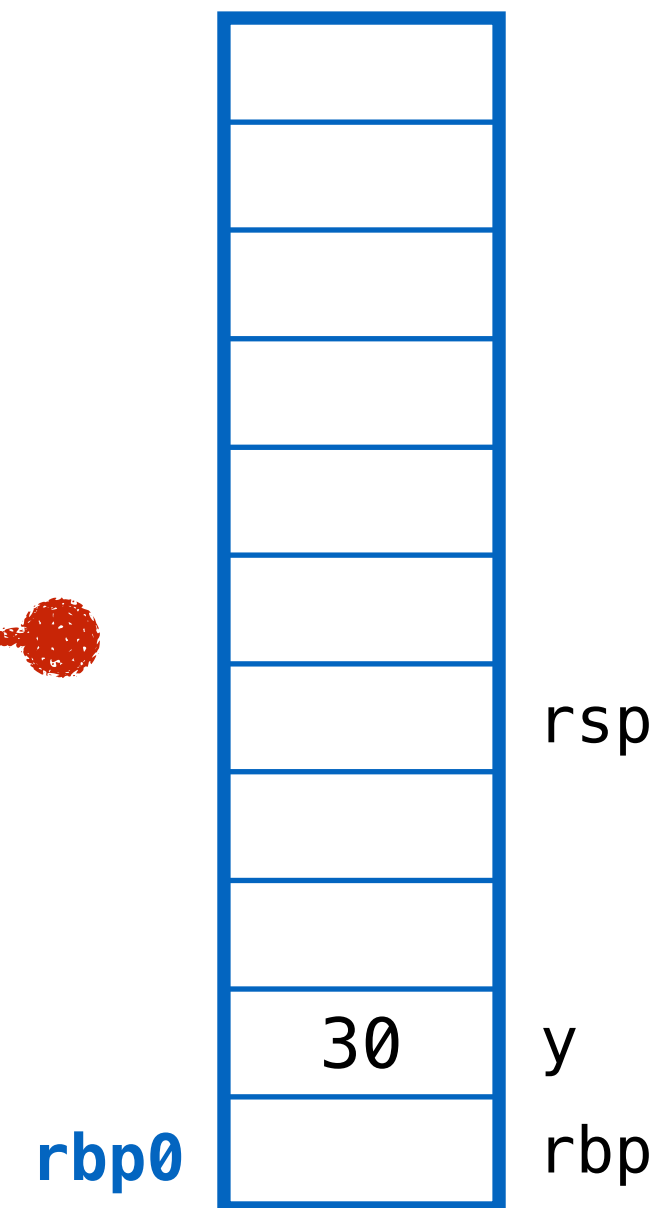


Return (rax) = 30



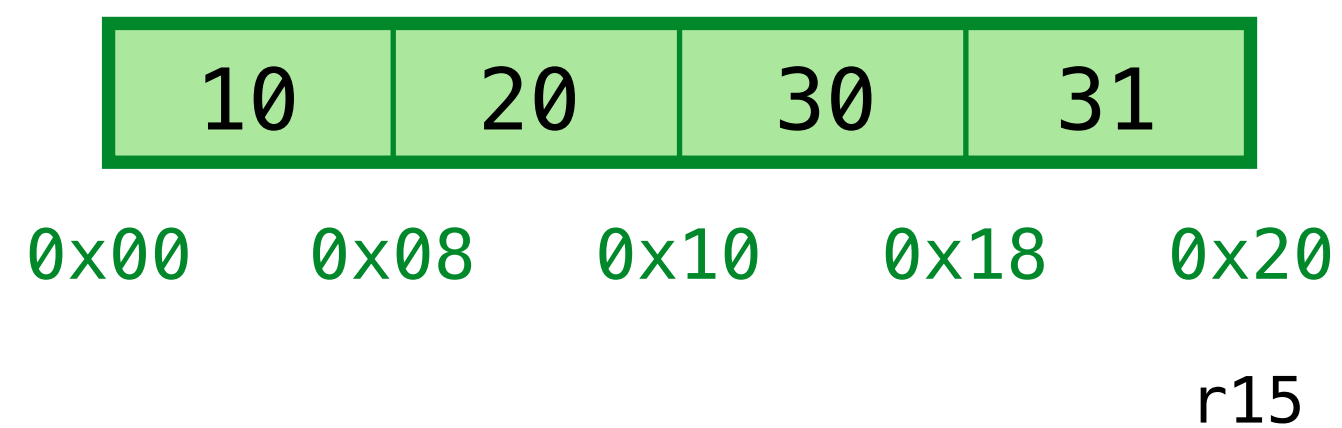
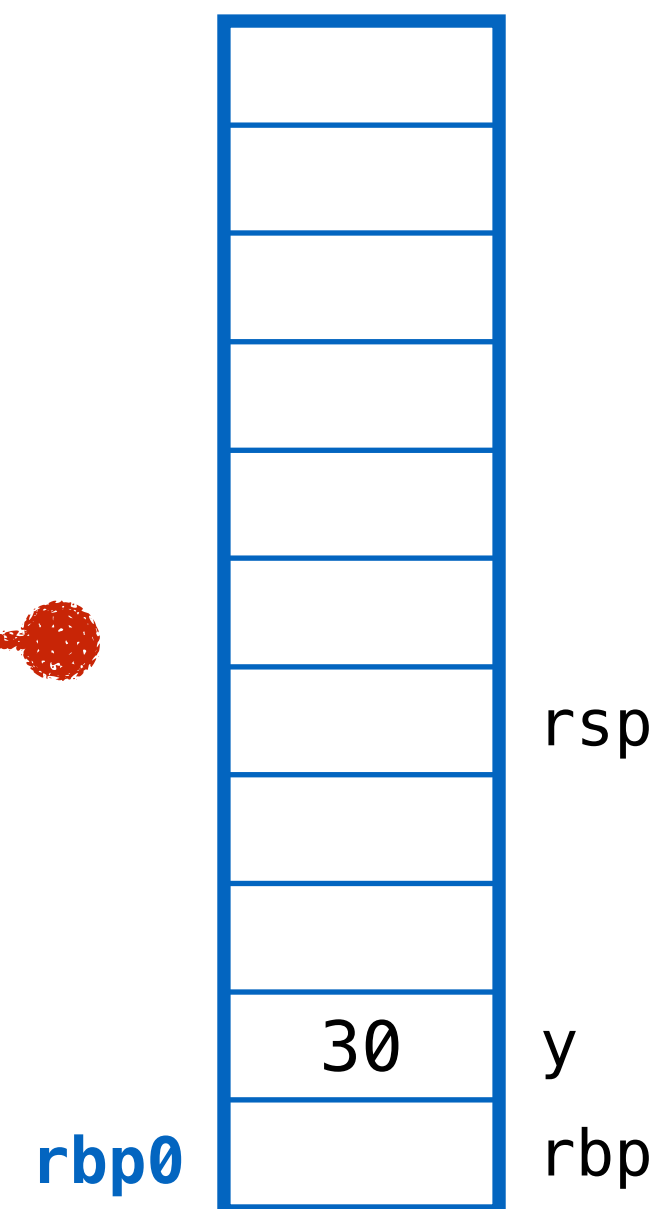
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



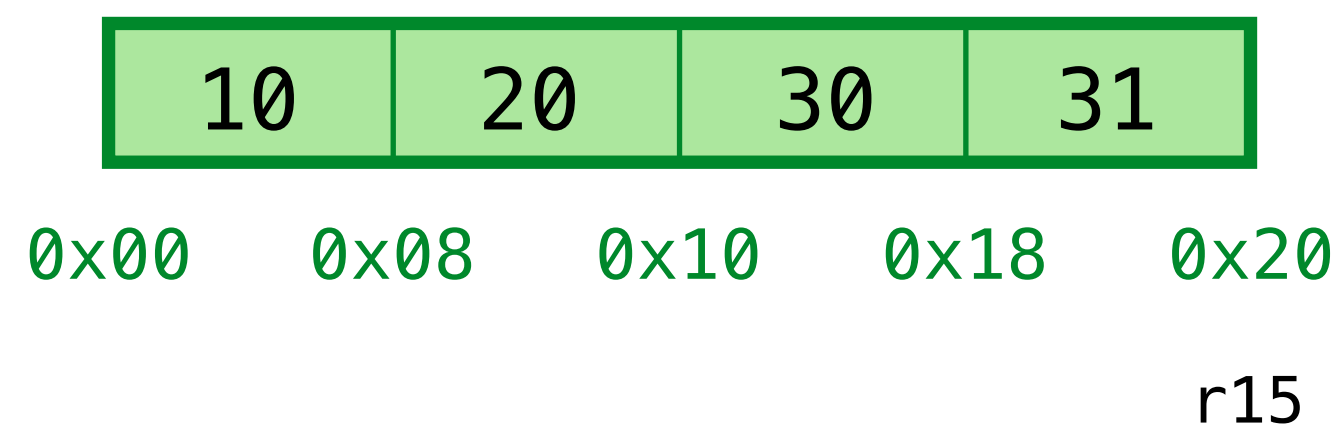
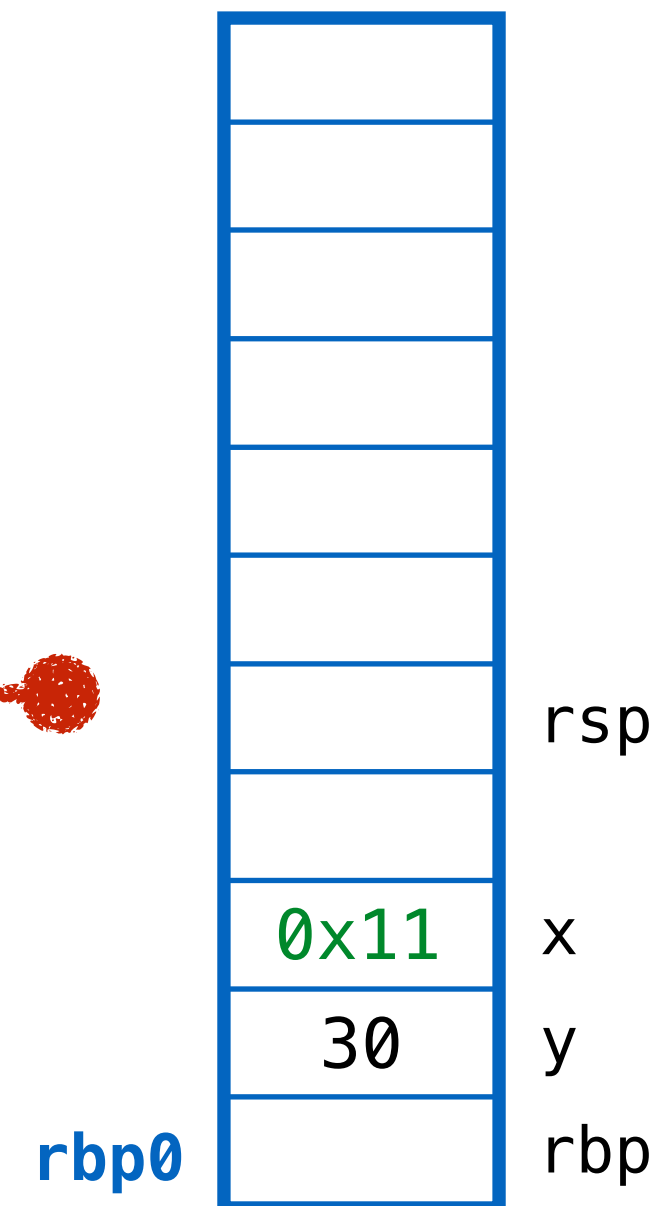
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



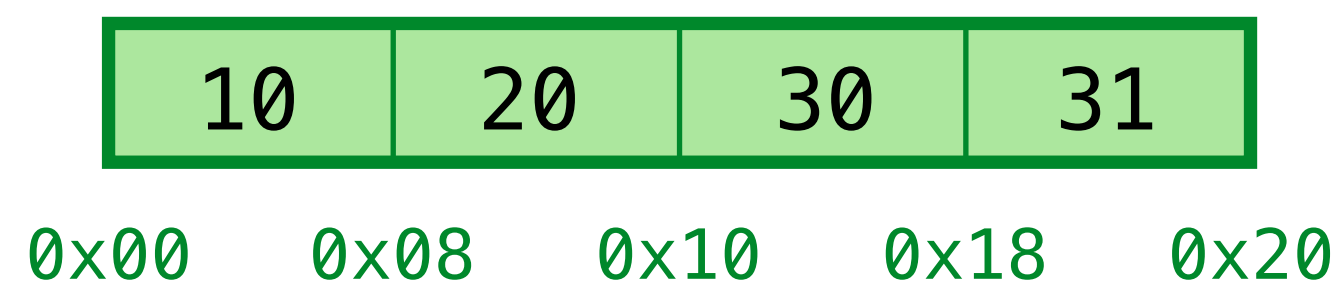
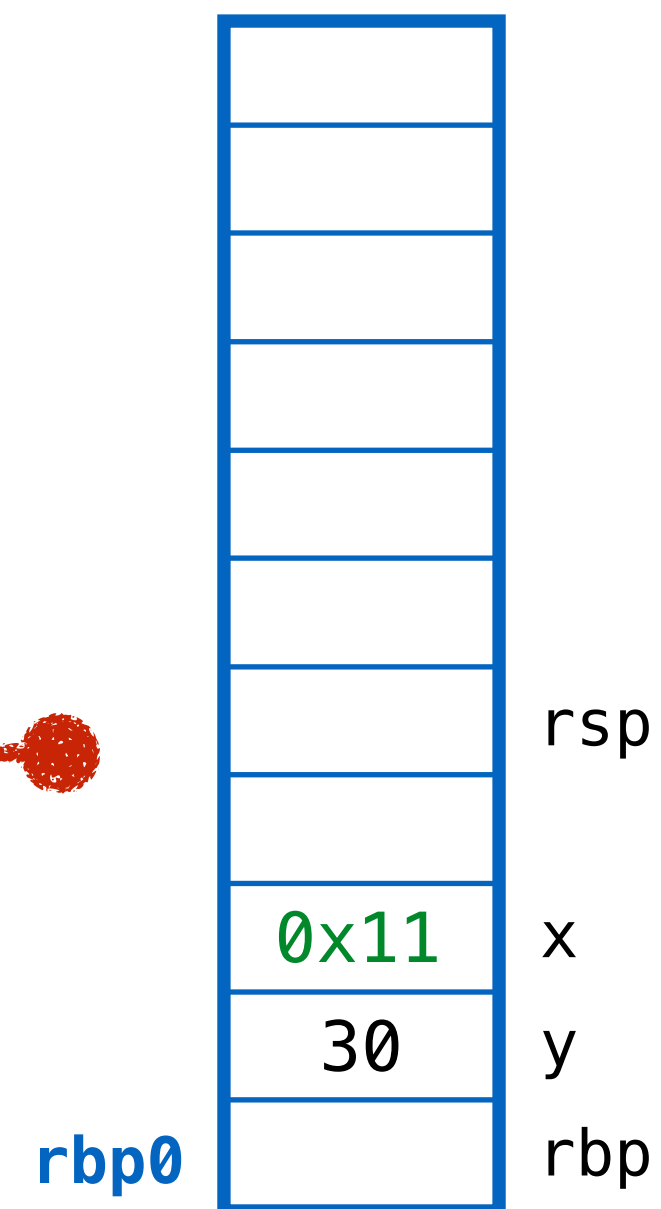
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
  , x = (y, y + 1)  
  , z = foo(100, 200)  
in  
  x[0] + z
```



ex3: garbage in the middle (with stack)

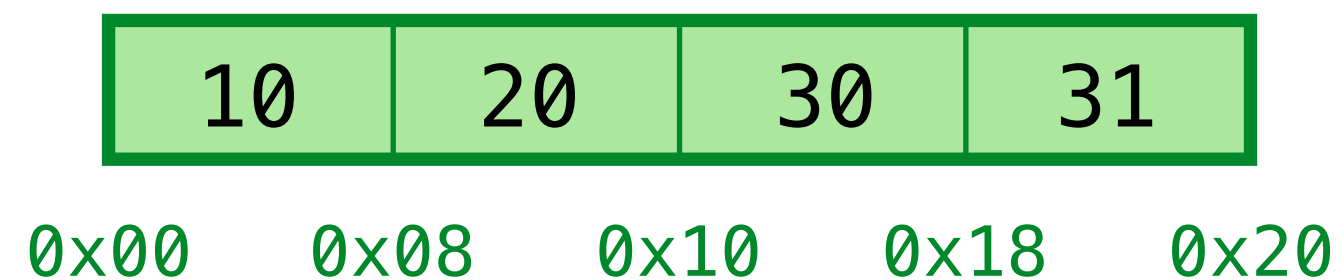
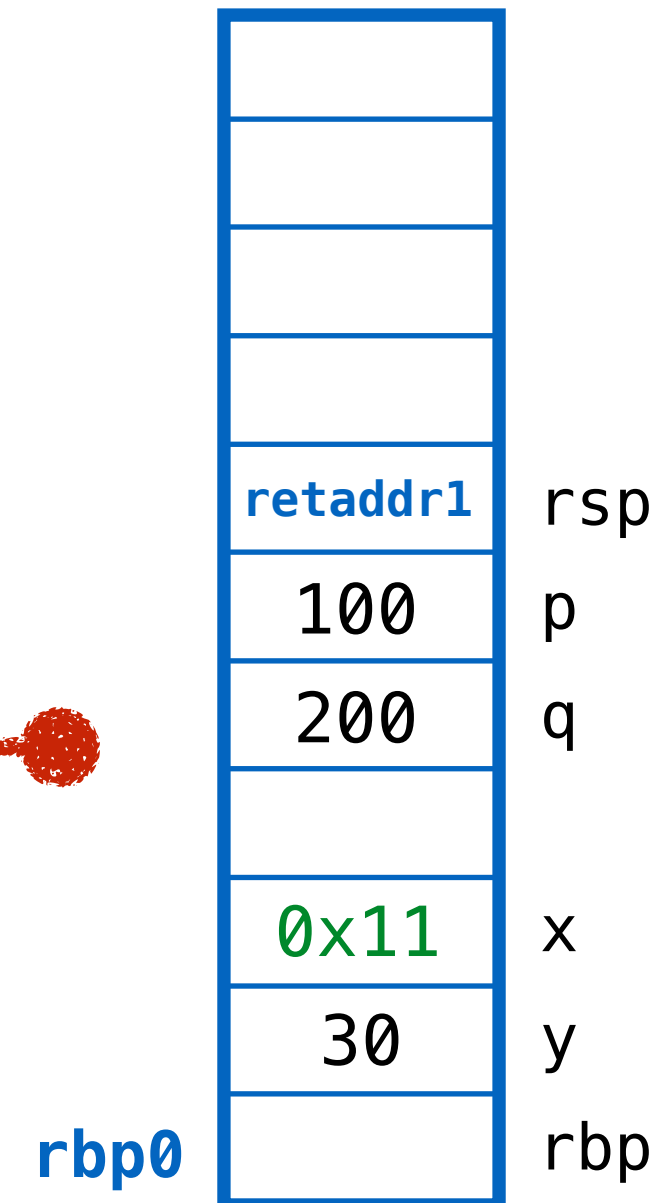
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



r15

ex3: garbage in the middle (with stack)

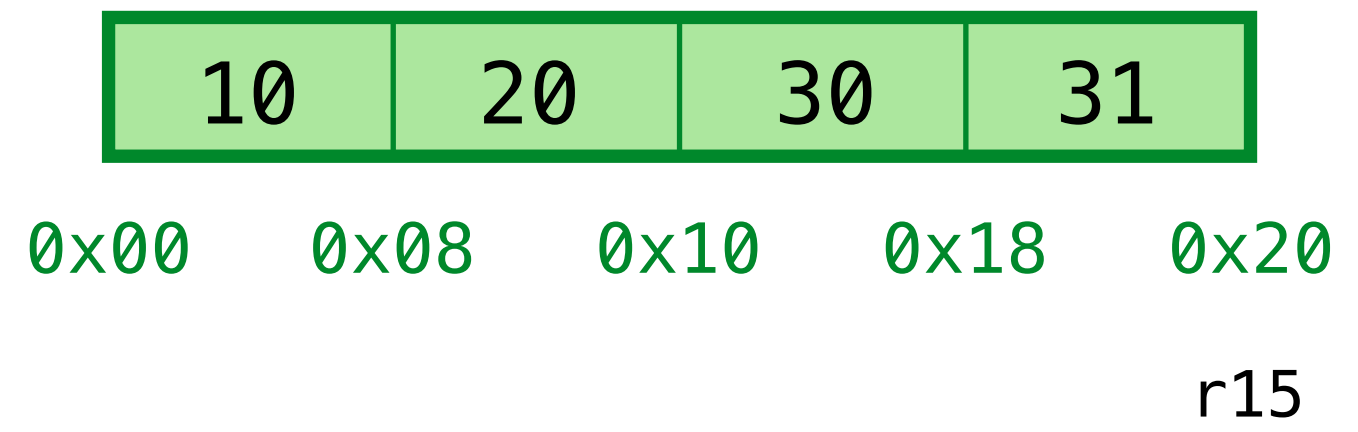
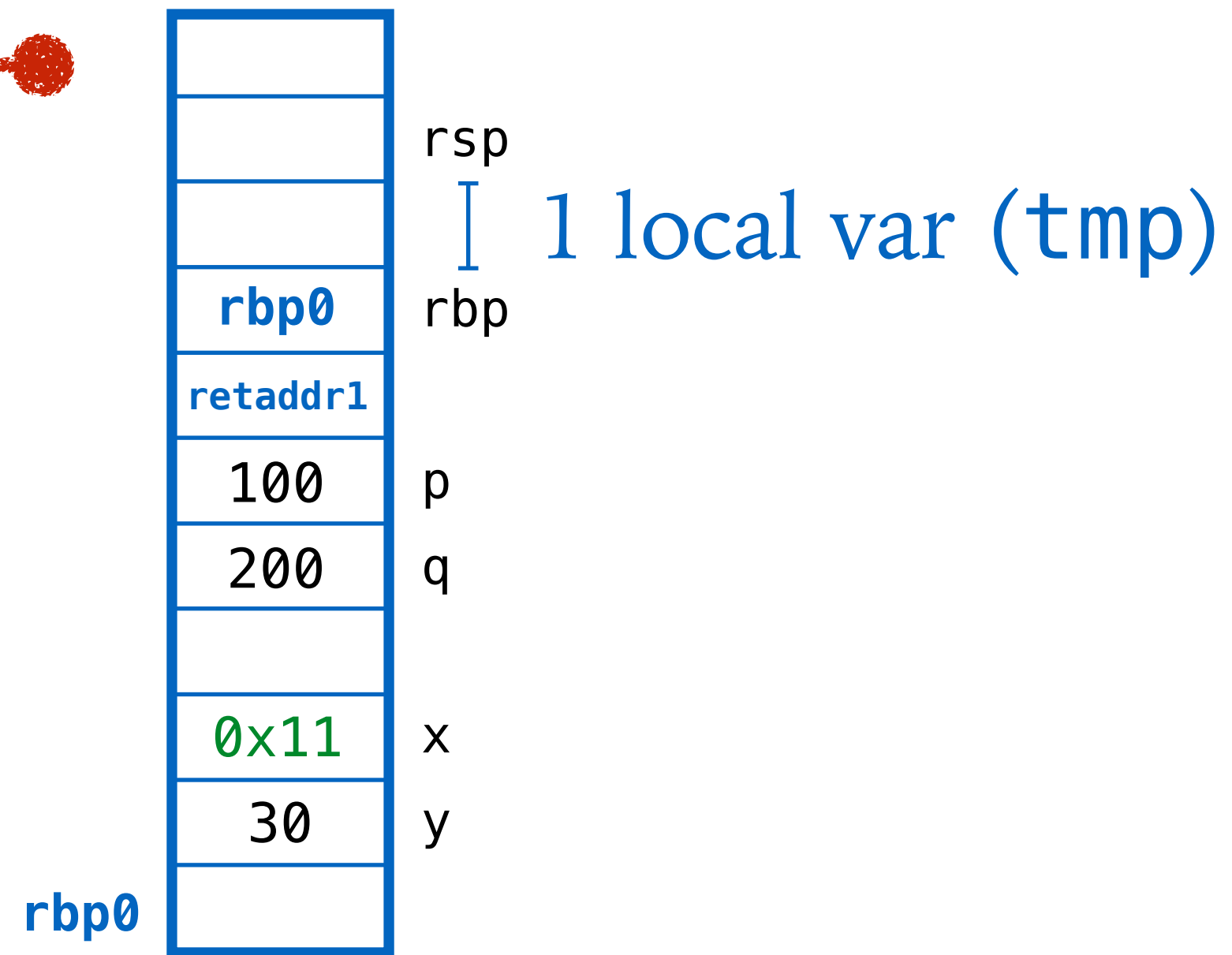
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



r15

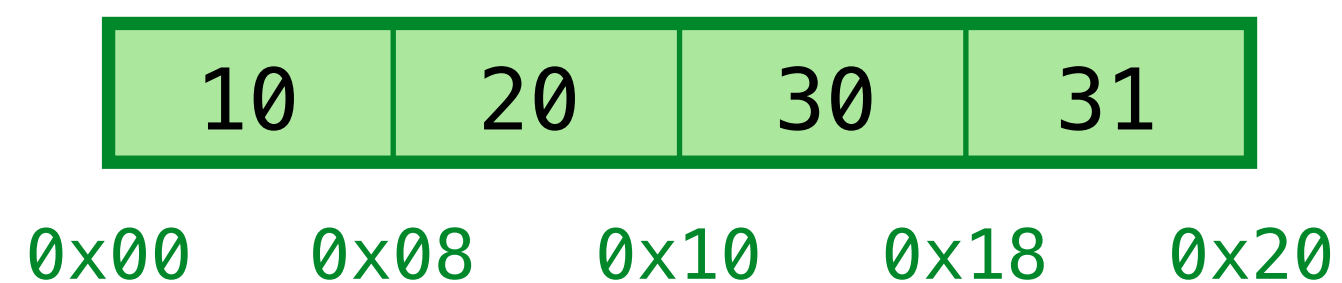
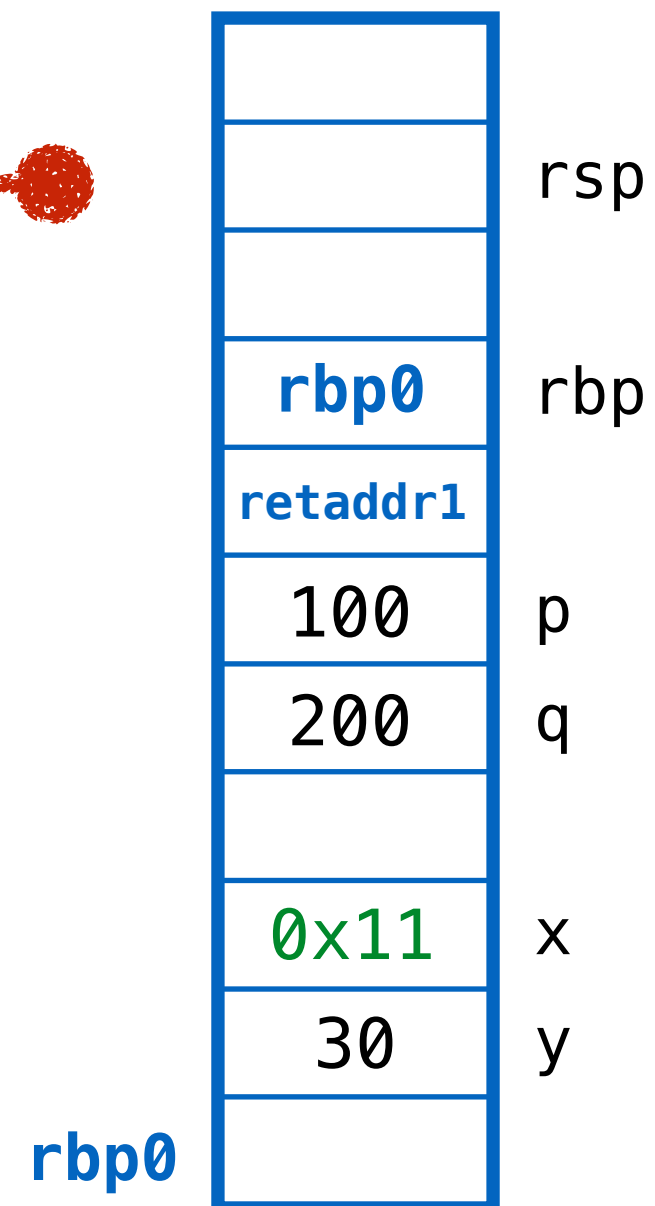
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



ex3: garbage in the middle (with stack)

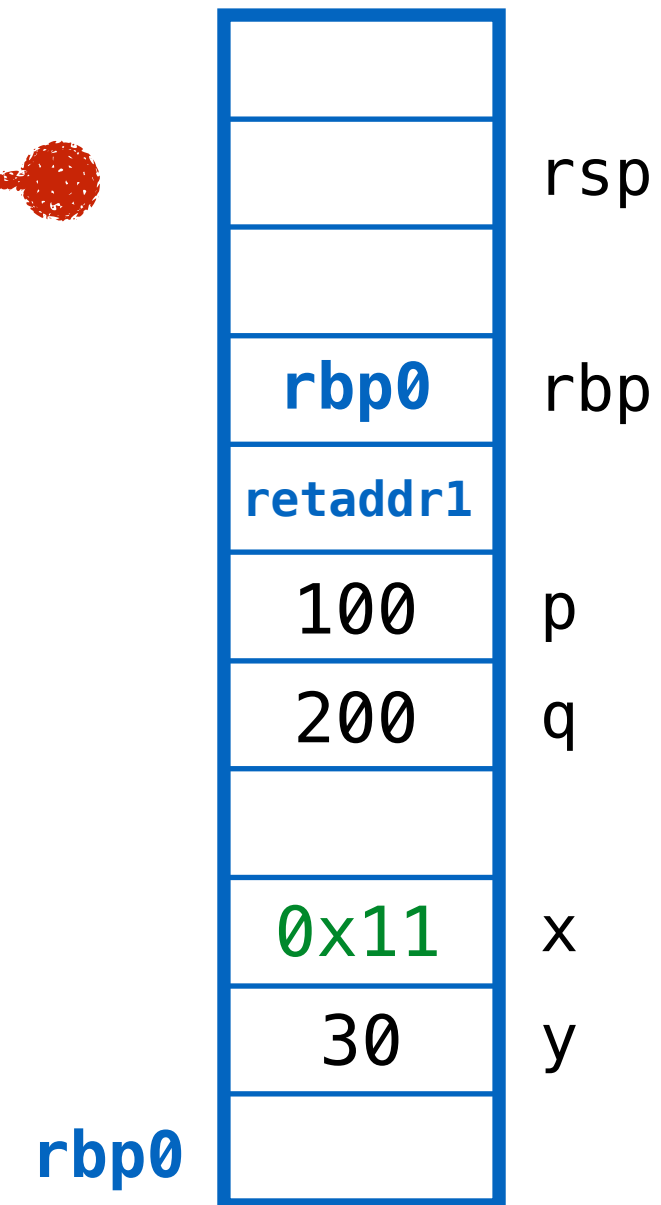
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



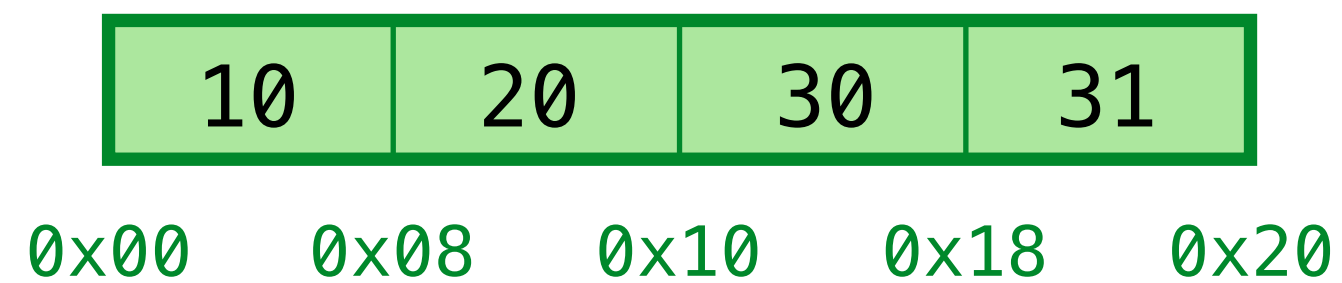
r15

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```

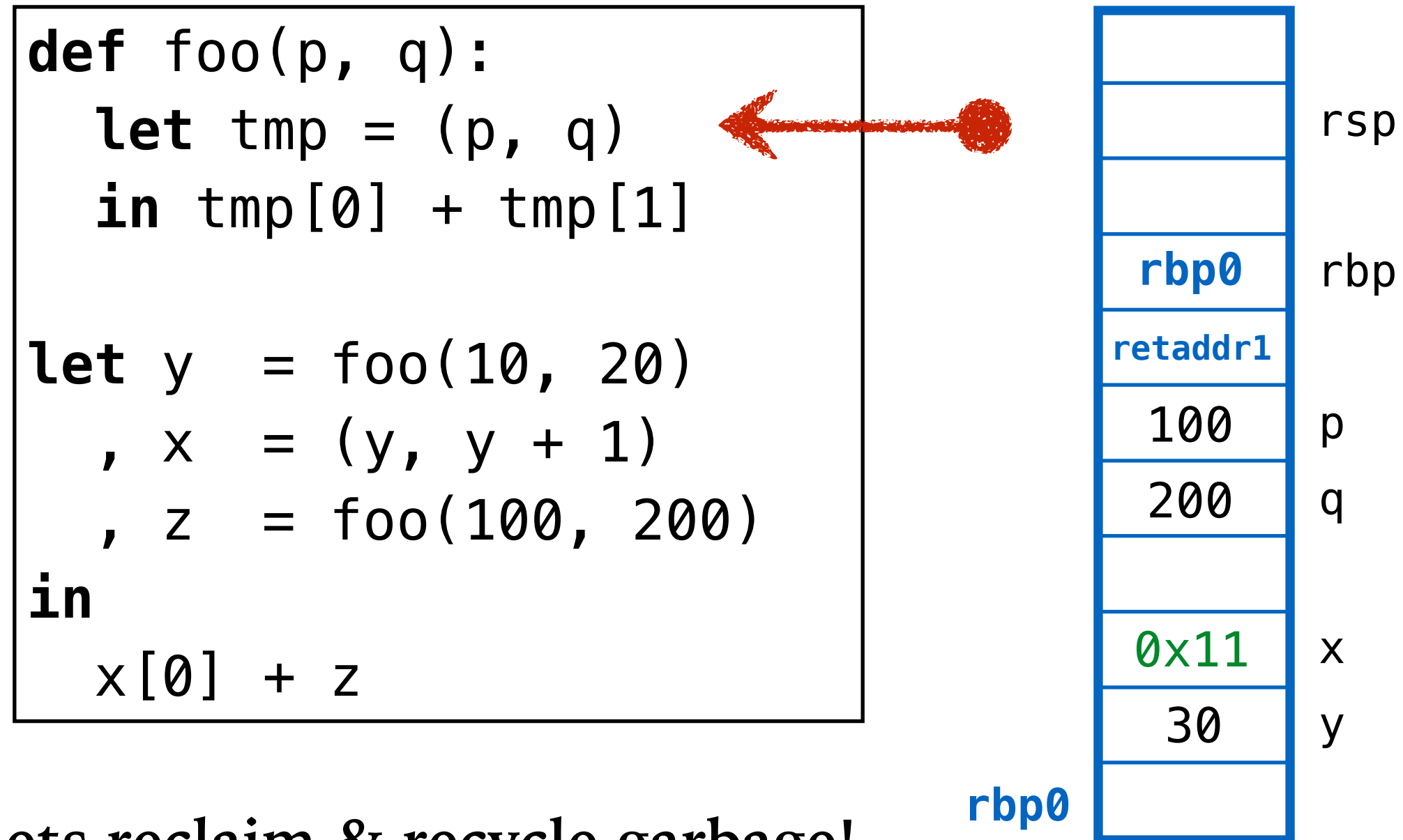


Lets reclaim & recycle garbage!

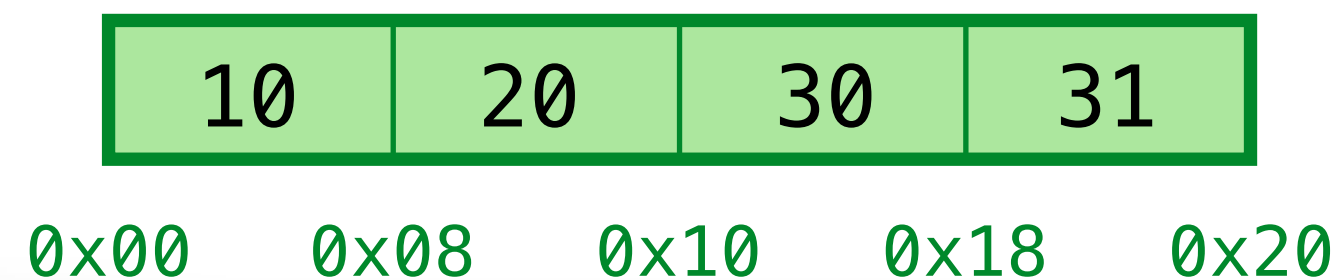


r15

ex3: garbage in the middle (with stack)



Lets reclaim & recycle garbage!

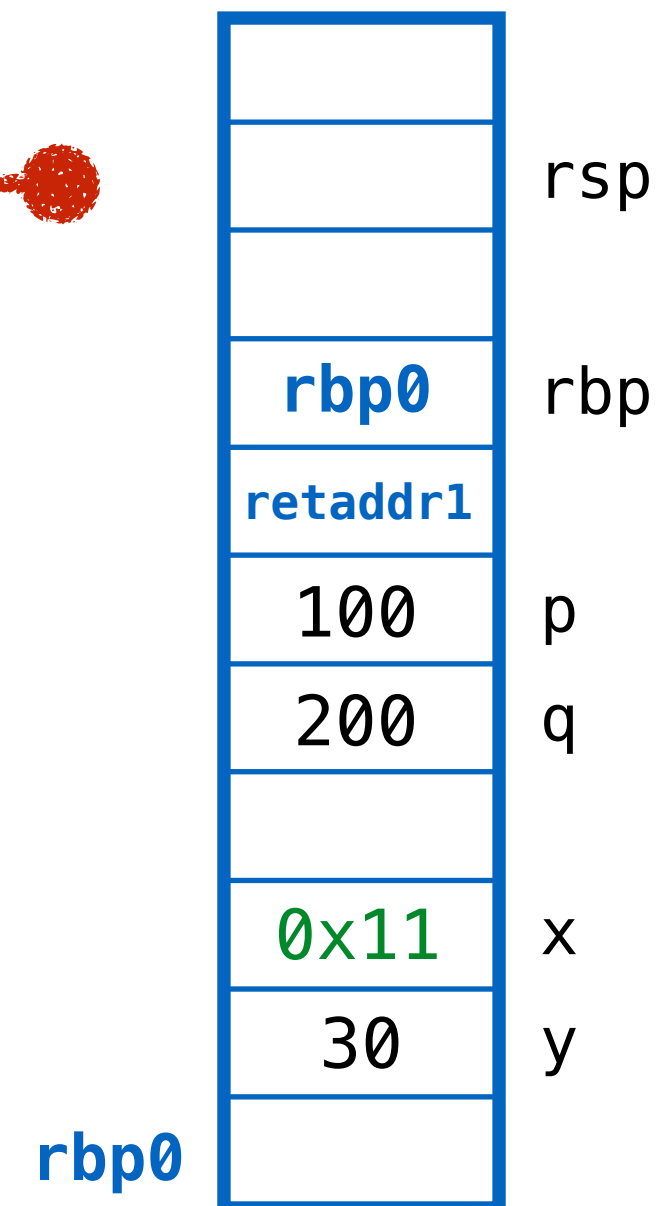


QUIZ: Which cells are garbage?

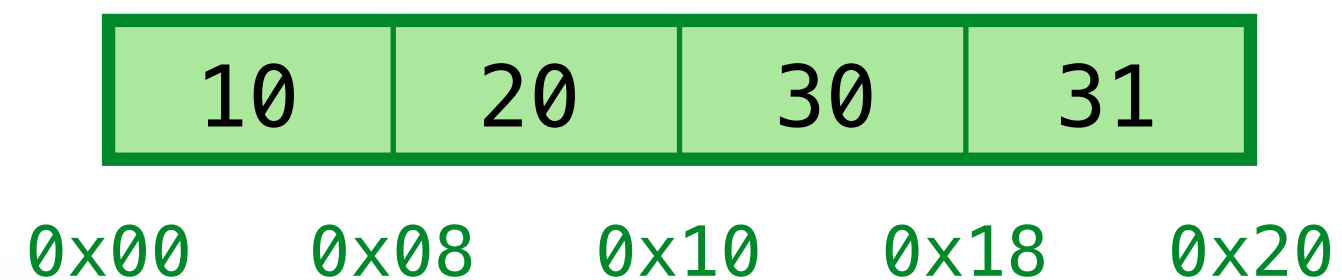
(A) 0x00, 0x08 (B) 0x08, 0x10 (C) 0x10, 0x18 (D) None (E) All

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
  let tmp = (p, q)  
  in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
  , x = (y, y + 1)  
  , z = foo(100, 200)  
in  
  x[0] + z
```



Lets reclaim & recycle garbage!

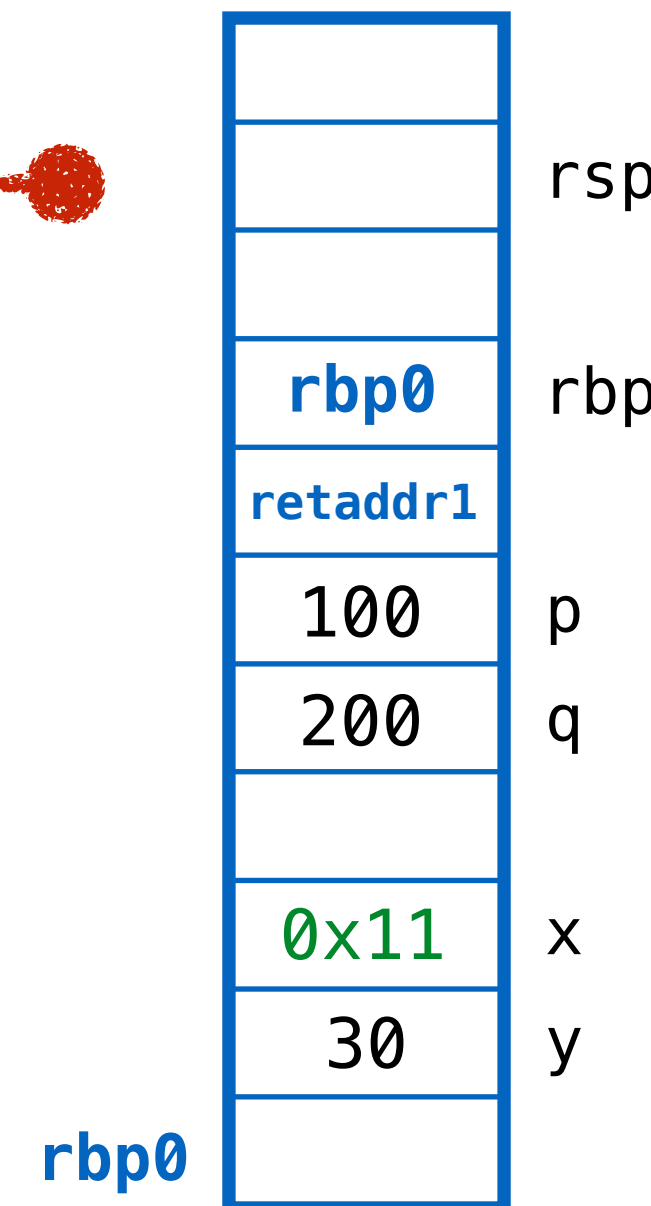


QUIZ: Which cells are garbage?

Those that are *not reachable from any stack frame*

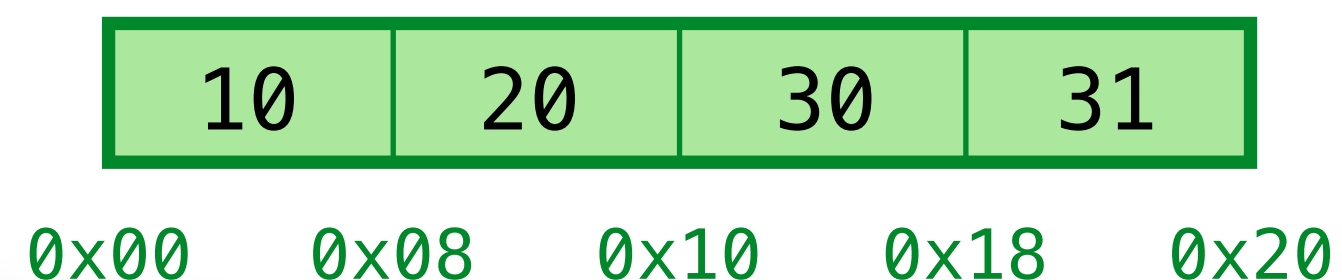
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
  let tmp = (p, q)  
  in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
  , x = (y, y + 1)  
  , z = foo(100, 200)  
in  
  x[0] + z
```



Traverse Stack
from top (rsp)
to bottom (rbp0)
to mark
reachable cells.

Lets reclaim & recycle garbage!

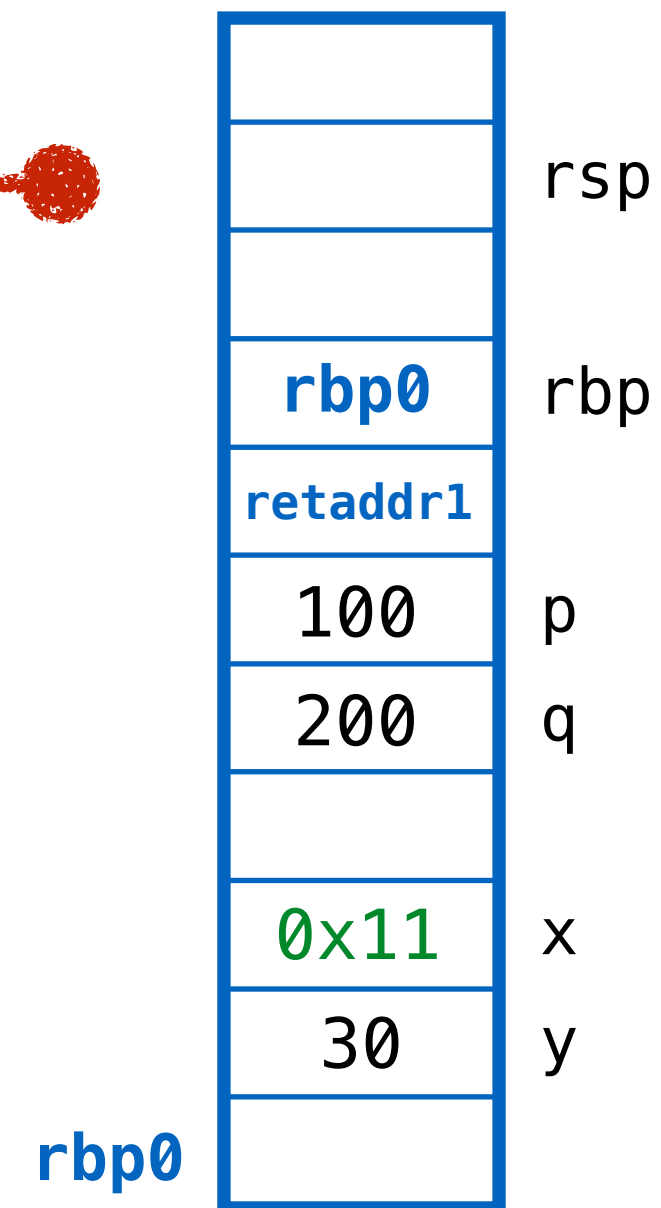


QUIZ: Which cells are garbage?

Those that are *not reachable from any stack frame*

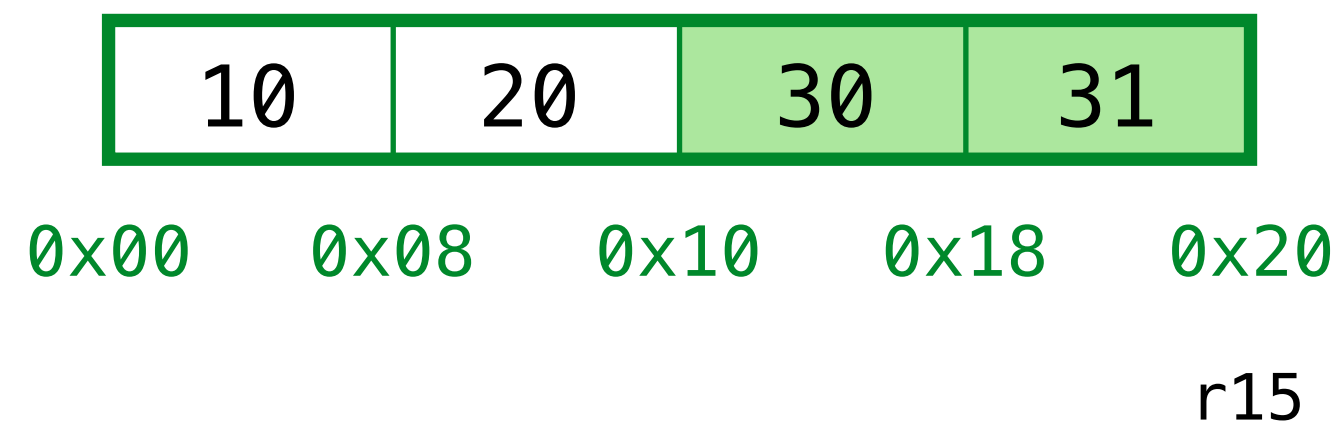
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
  let tmp = (p, q)  
  in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
  , x = (y, y + 1)  
  , z = foo(100, 200)  
in  
  x[0] + z
```



Traverse Stack
from top (rsp)
to bottom (rbp0)
to mark
reachable cells.

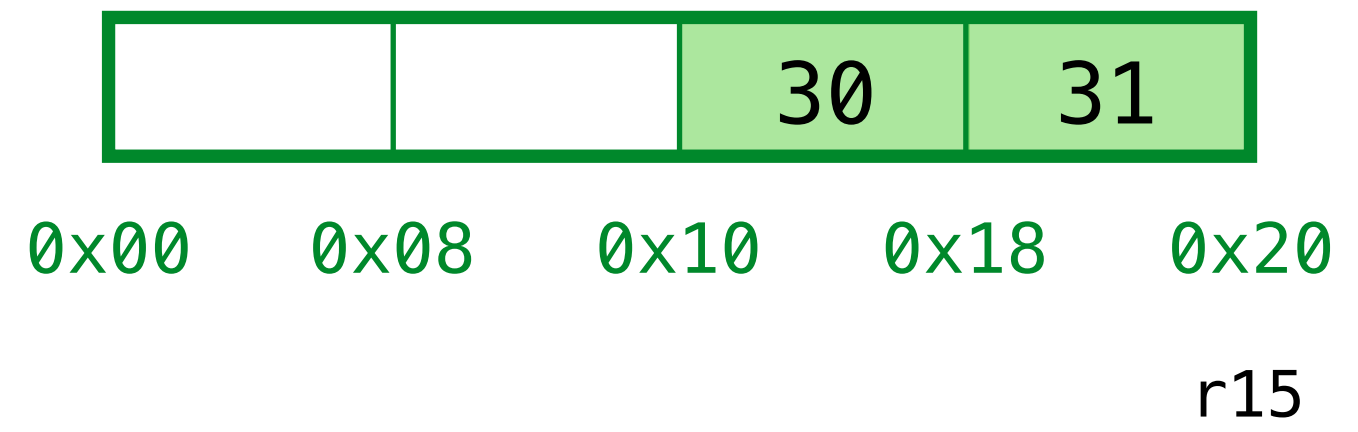
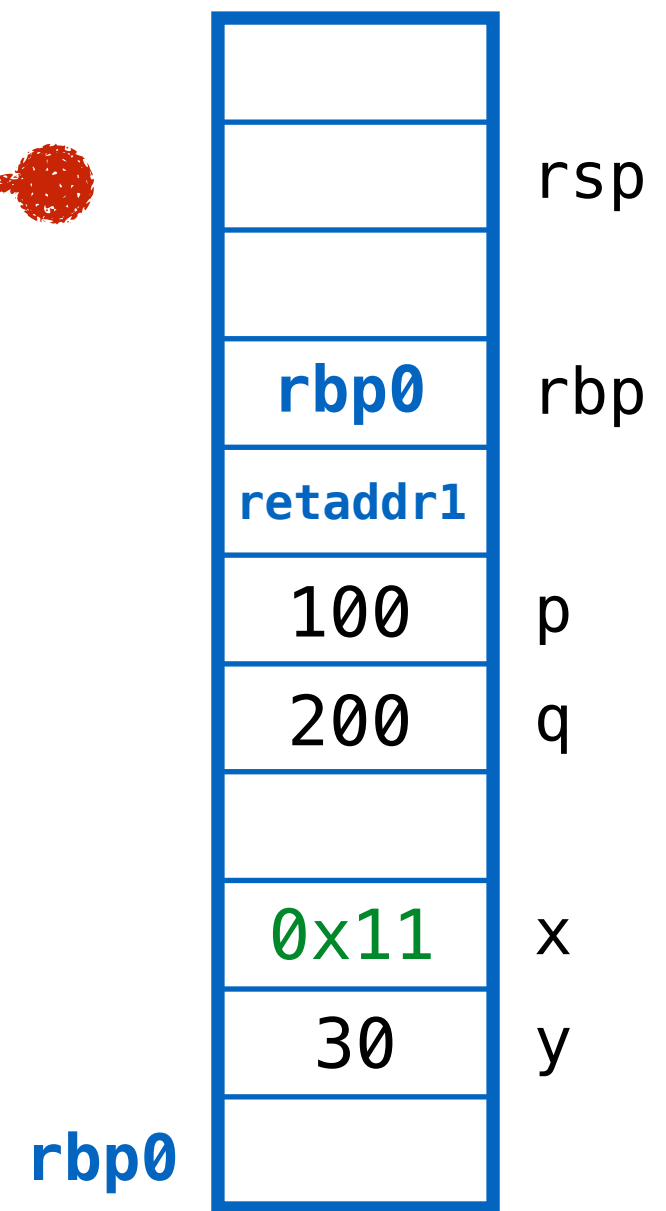
Lets reclaim & recycle garbage!



Which cells are garbage?

ex3: garbage in the middle (with stack)

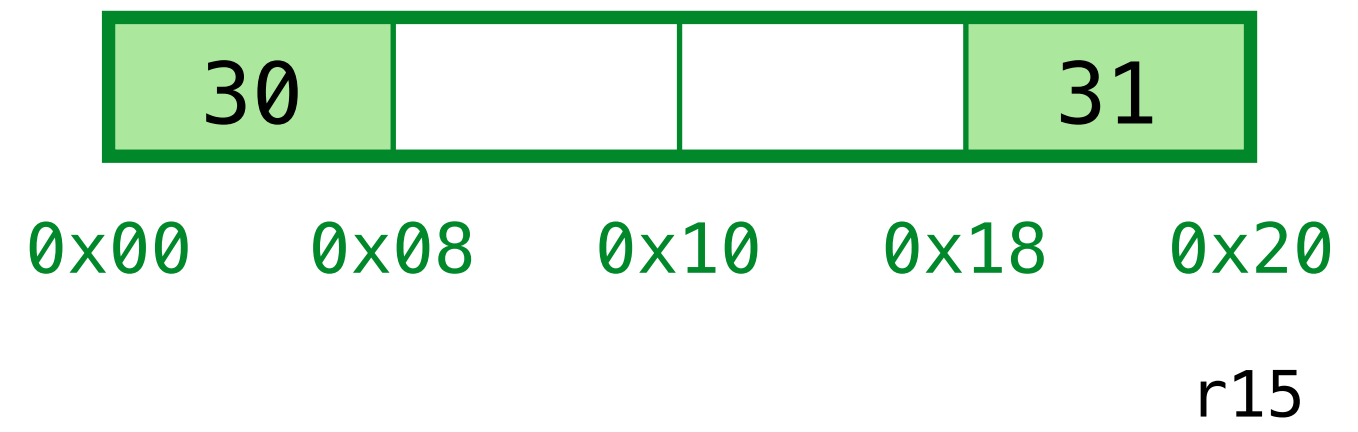
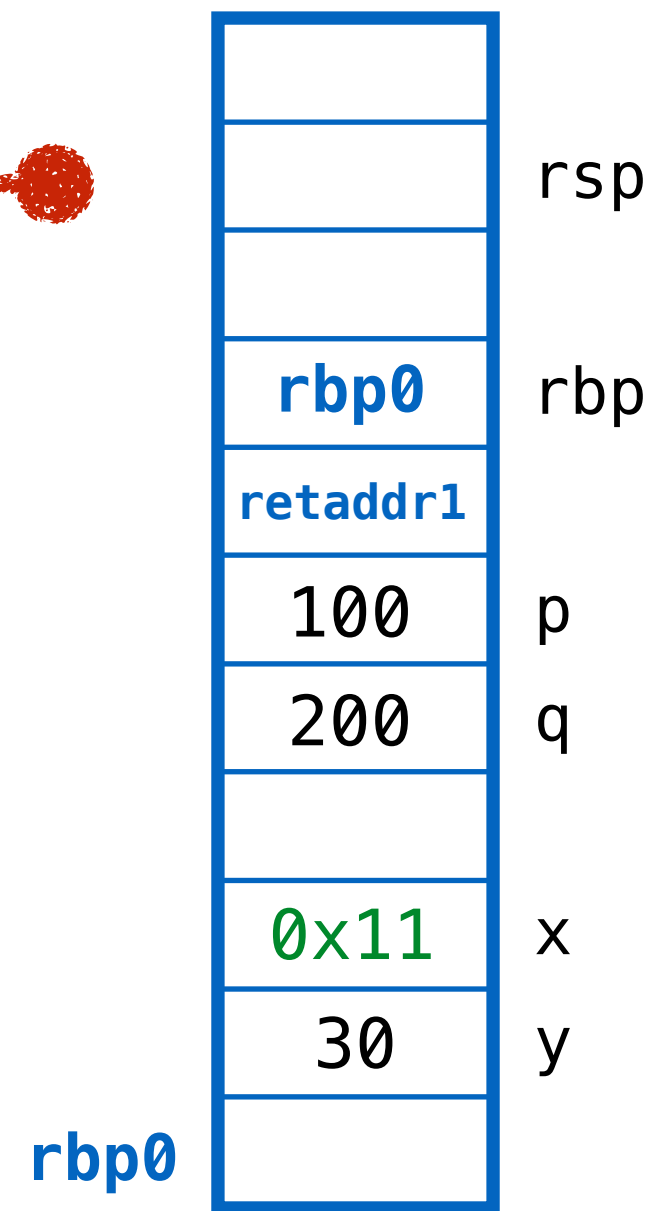
```
def foo(p, q):  
  let tmp = (p, q)  
  in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
  , x = (y, y + 1)  
  , z = foo(100, 200)  
in  
  x[0] + z
```



Compact the live cells

ex3: garbage in the middle (with stack)

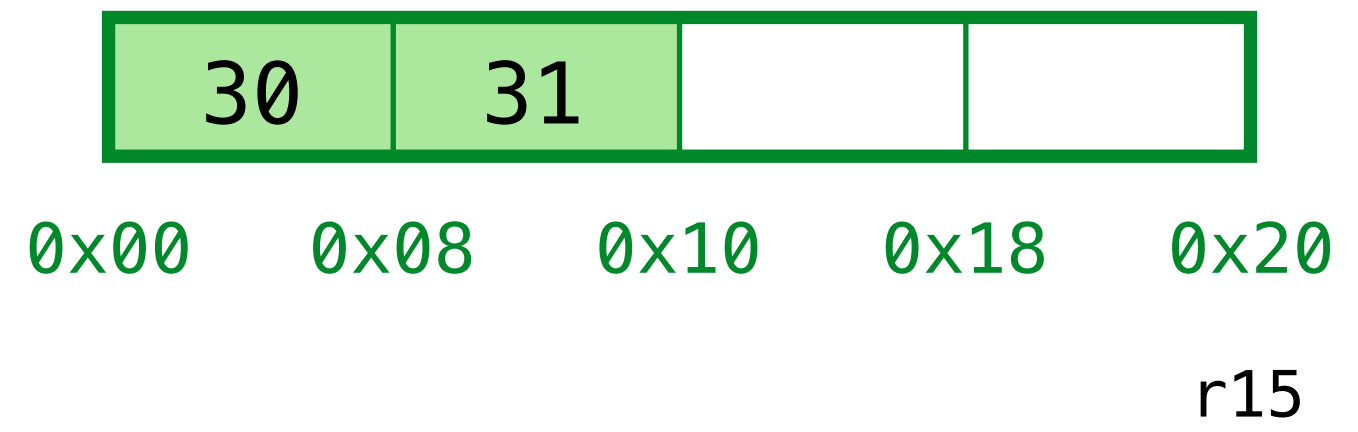
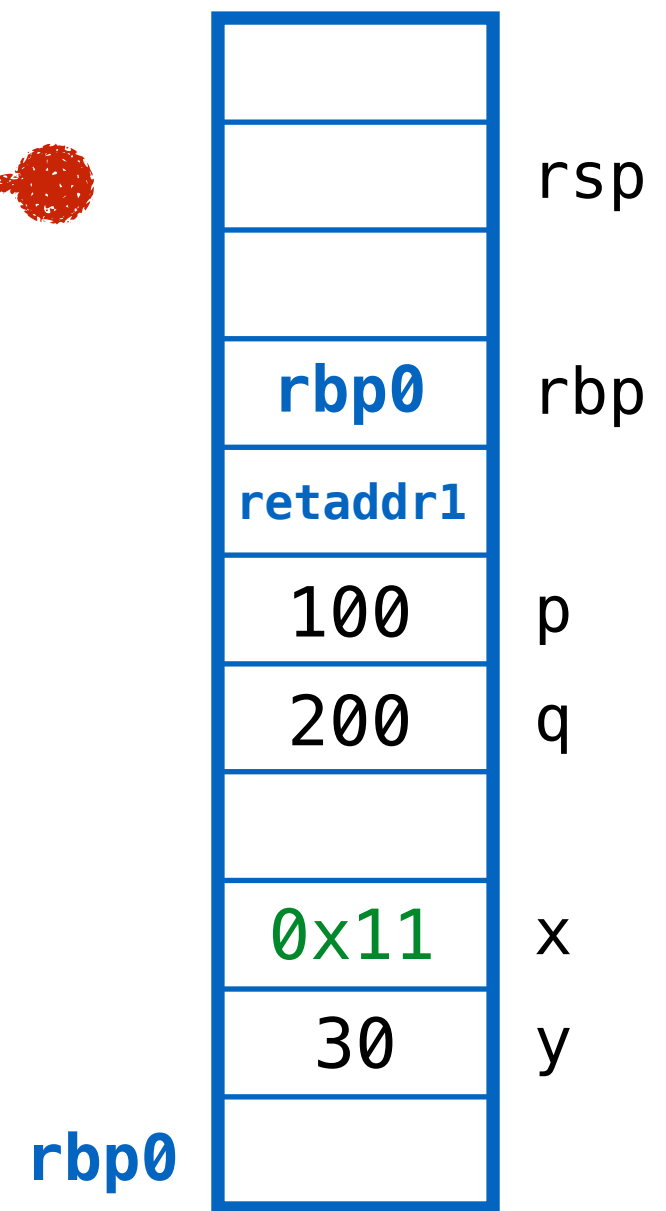
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



Compact the live cells

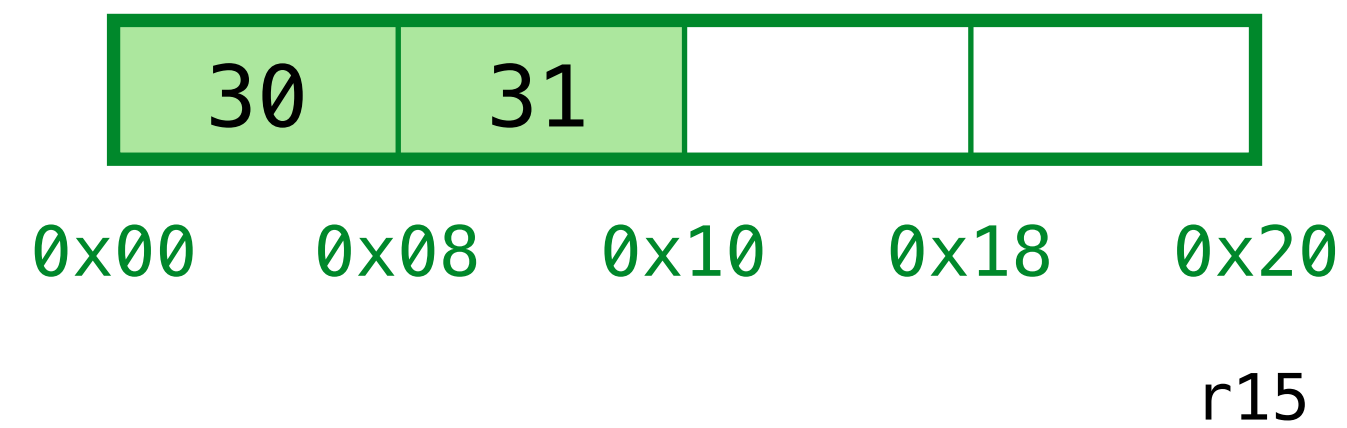
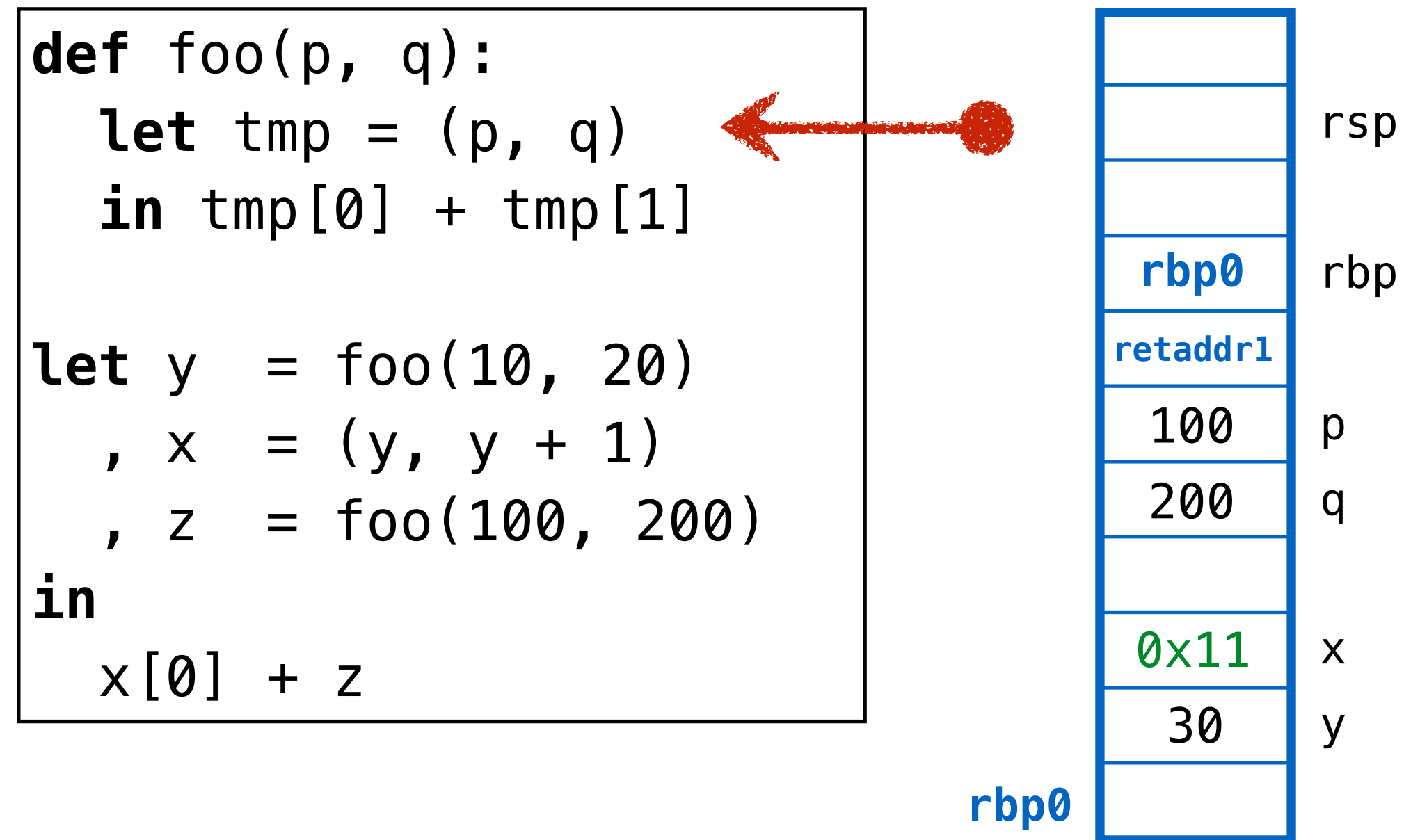
ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



Compact the live cells

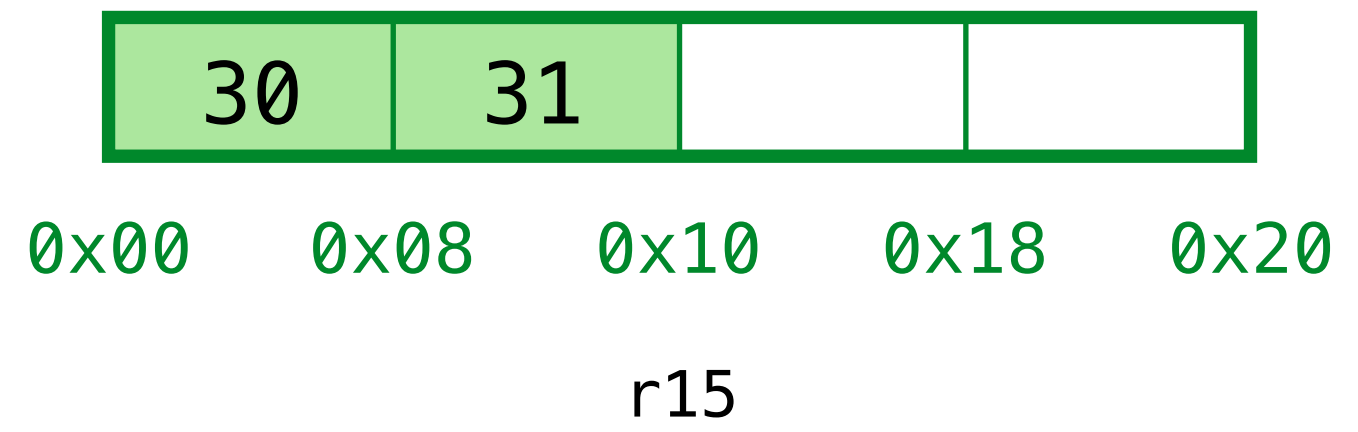
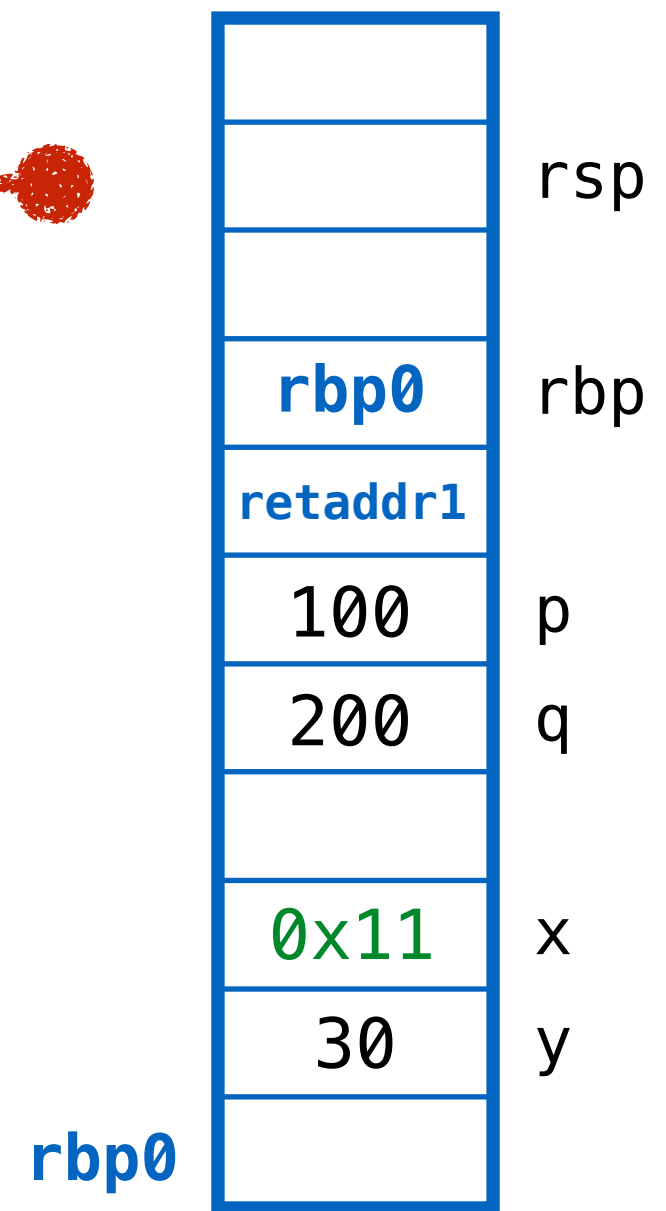
ex3: garbage in the middle (with stack)



Compact the live cells ... then rewind r15

ex3: garbage in the middle (with stack)

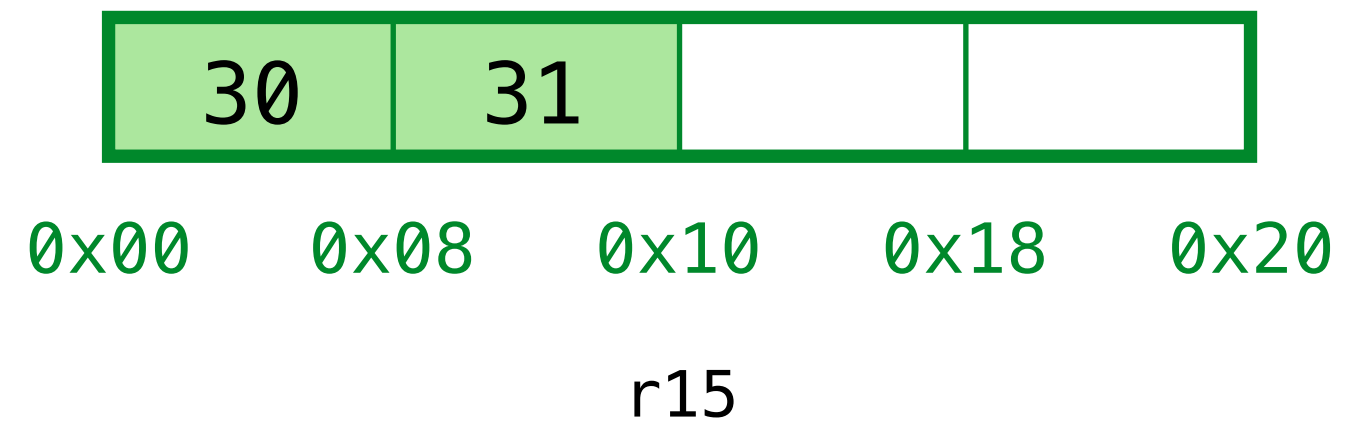
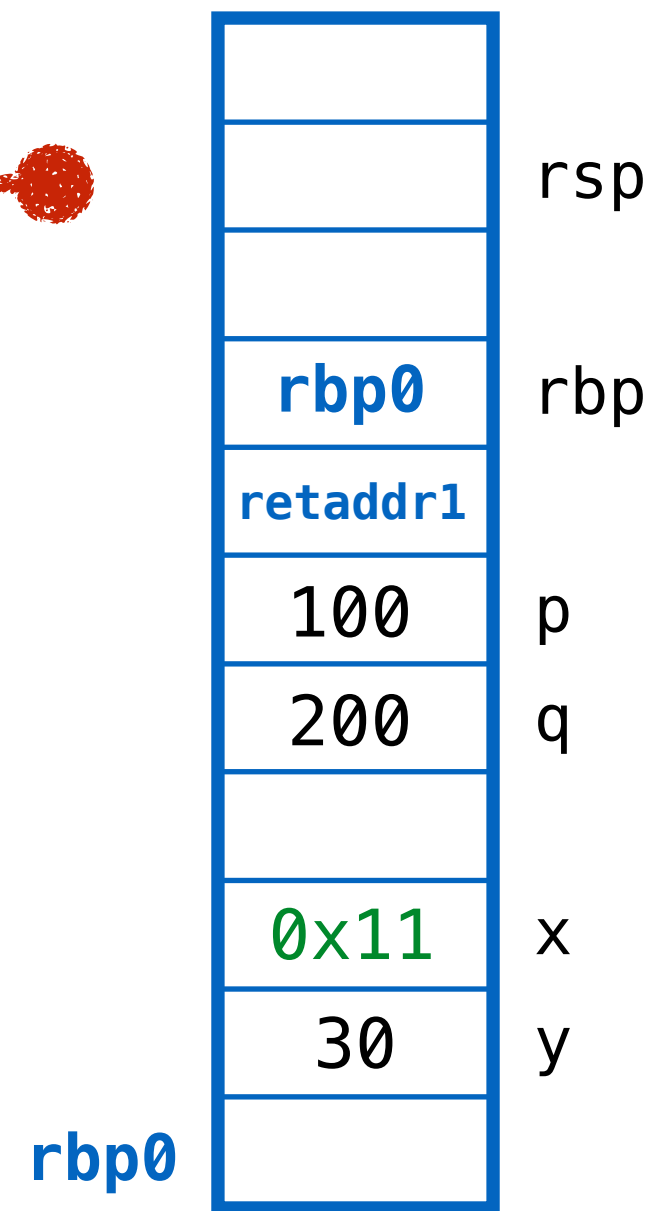
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



Compact the live cells ... then rewind r15

ex3: garbage in the middle (with stack)

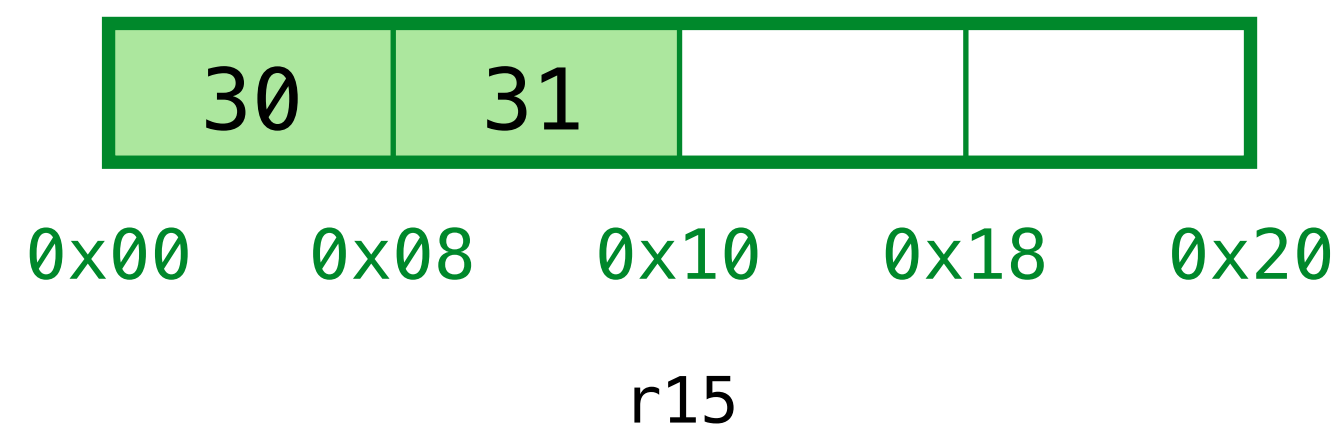
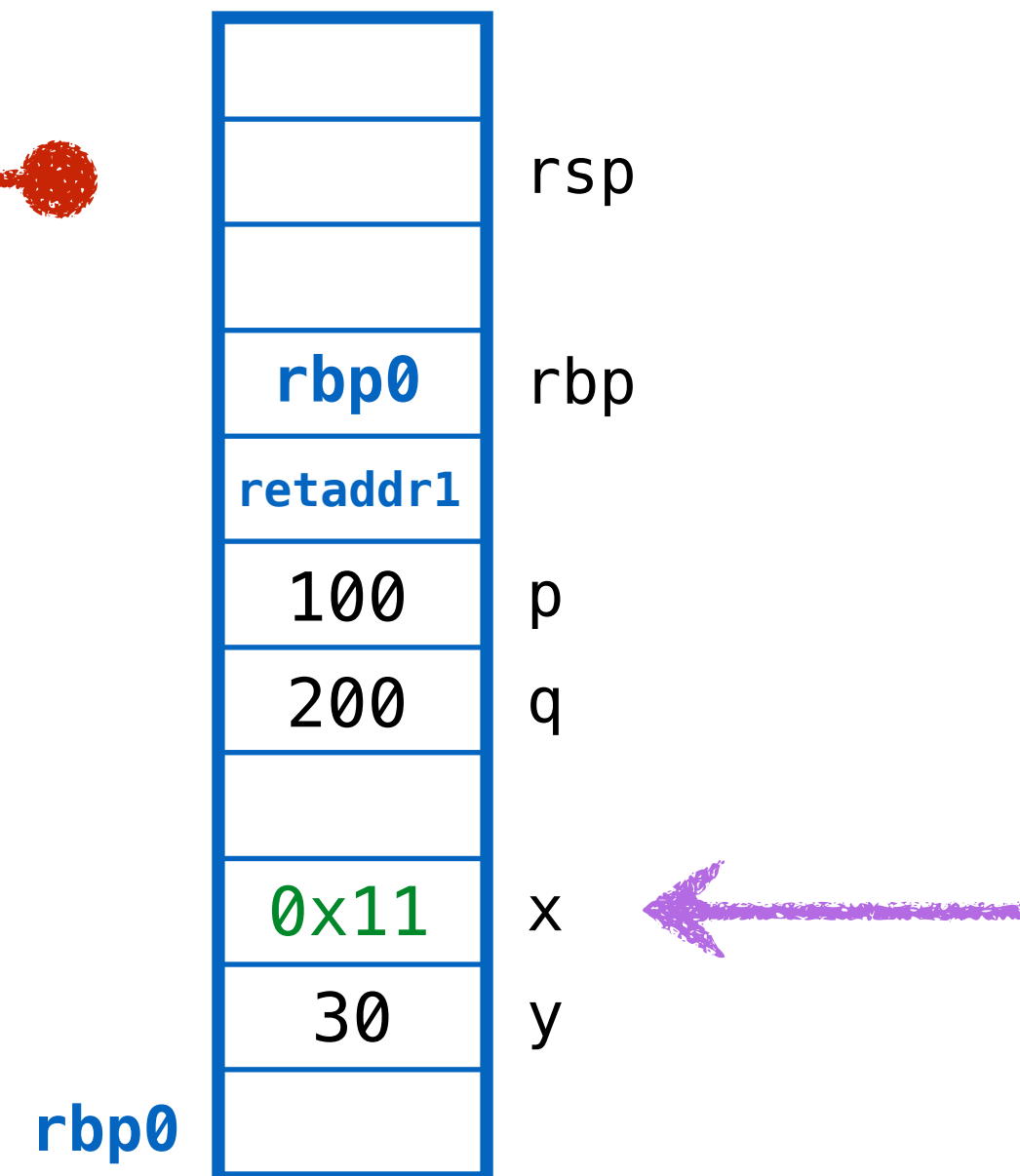
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



Problem???

ex3: garbage in the middle (with stack)

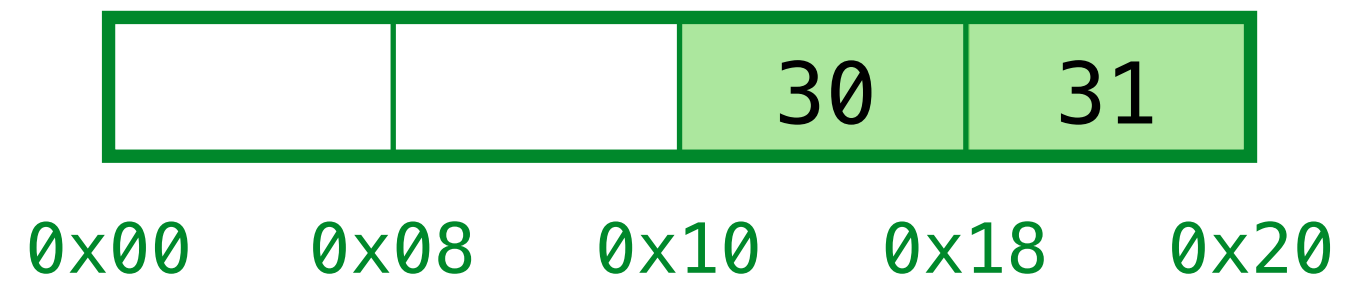
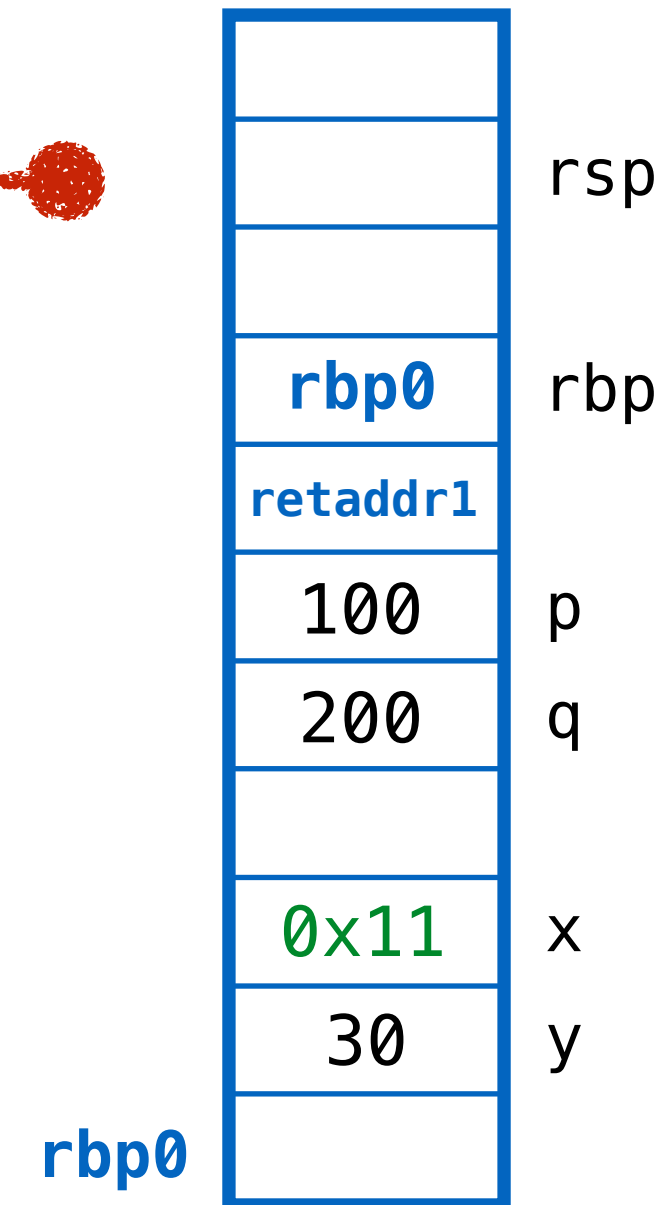
```
def foo(p, q):  
  let tmp = (p, q)  
  in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
  , x = (y, y + 1)  
  , z = foo(100, 200)  
in  
  x[0] + z
```



Problem! Have to REDIRECT existing pointers

ex3: garbage in the middle (with stack)

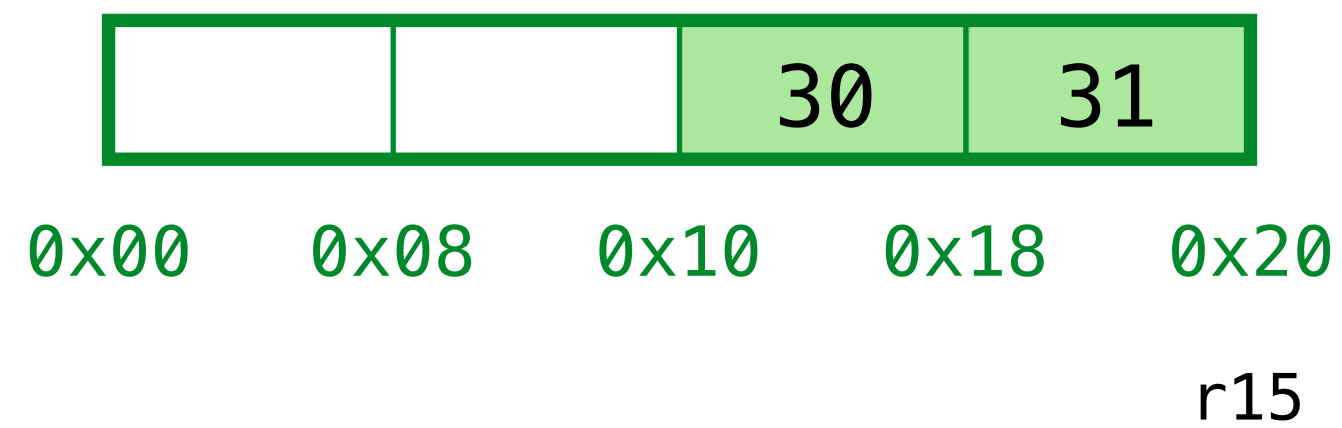
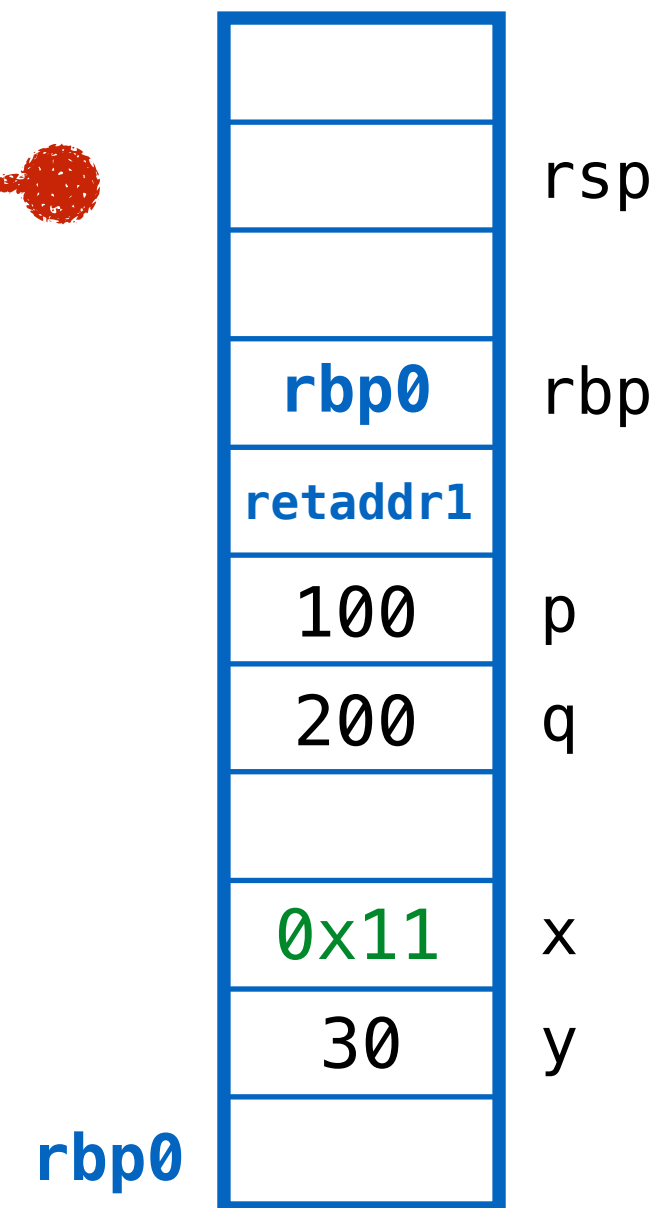
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



r15

ex3: garbage in the middle (with stack)

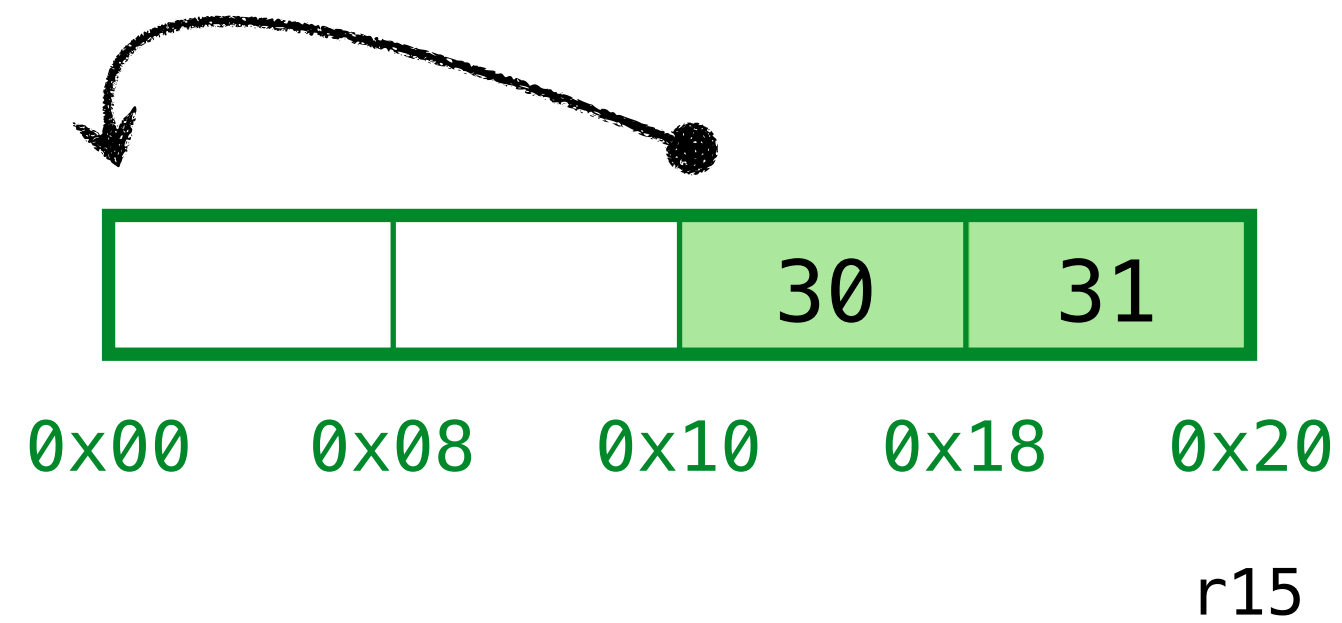
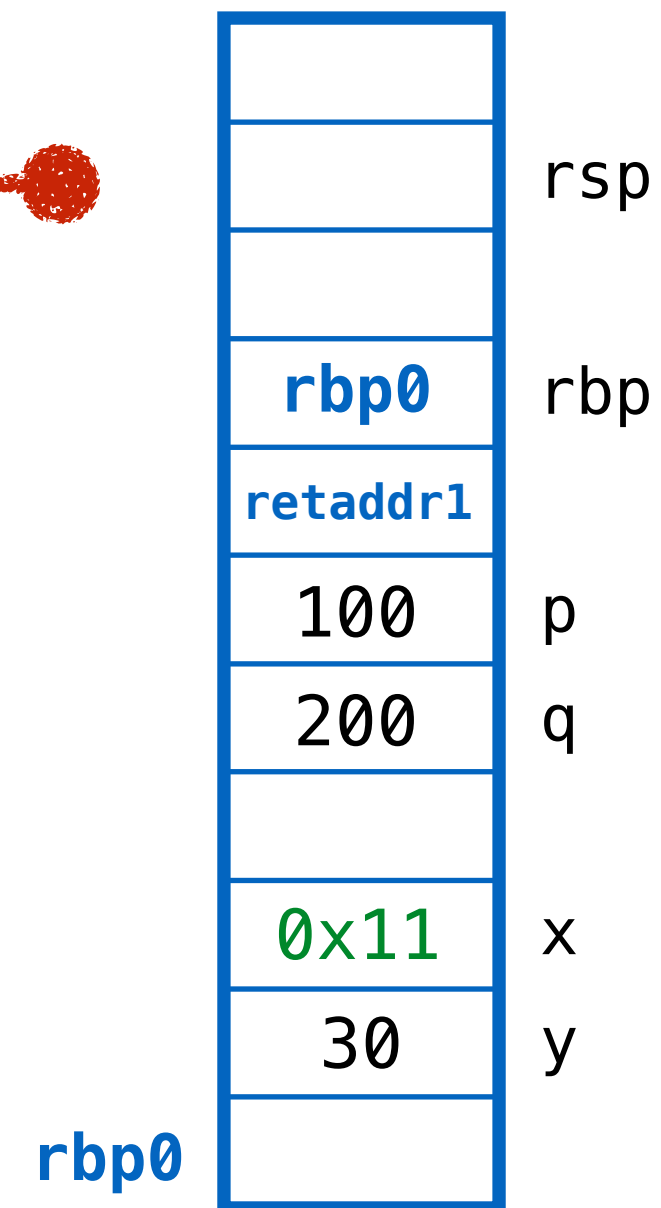
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



1. Compute **FORWARD** addrs (i.e. new compacted addrs)

ex3: garbage in the middle (with stack)

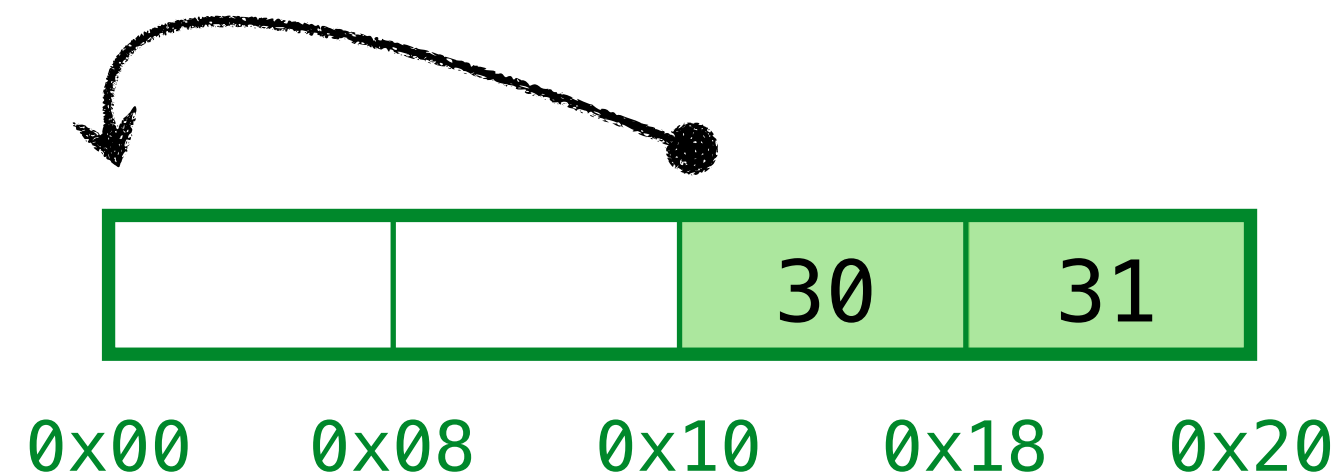
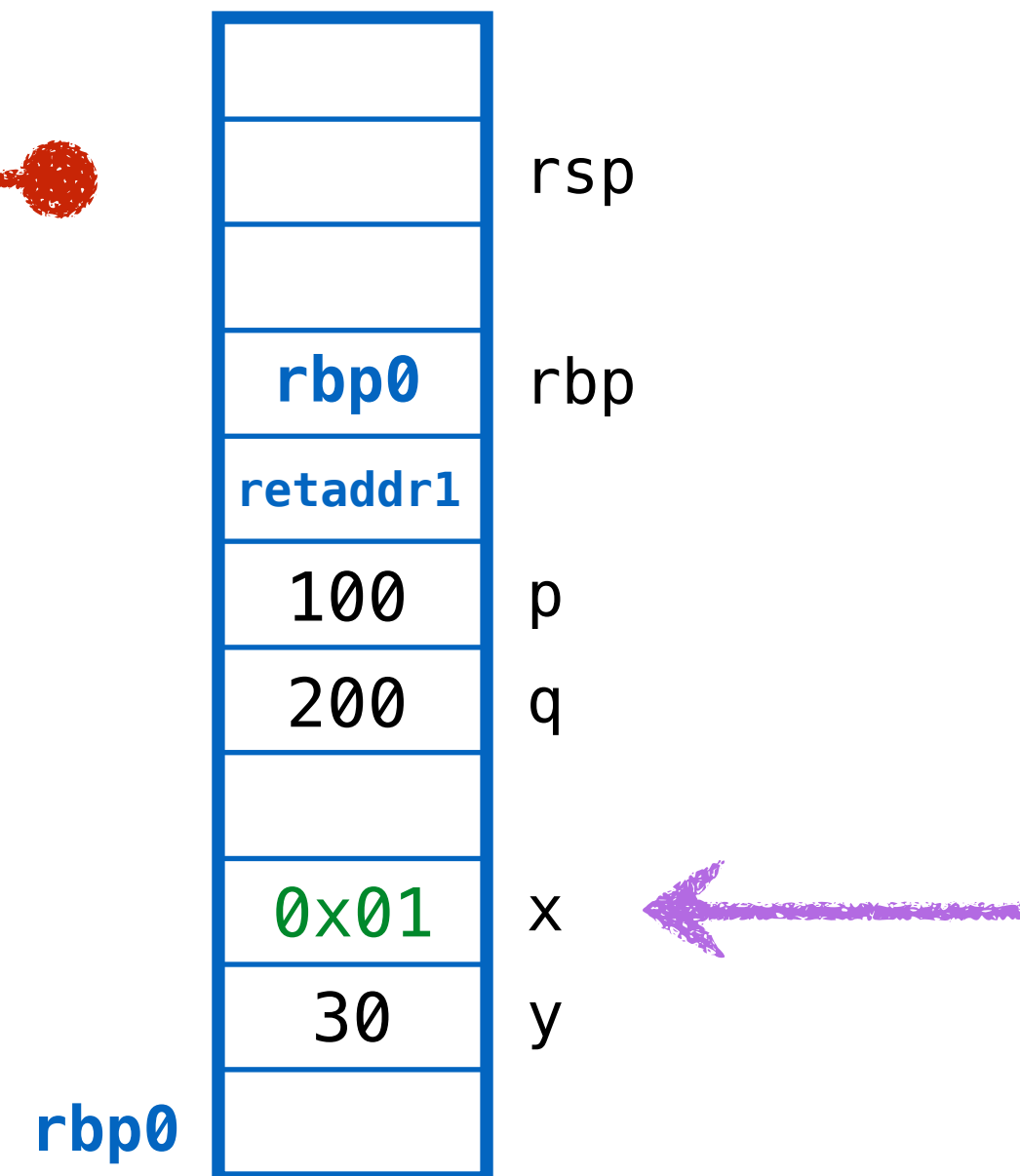
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



1. Compute **FORWARD** addrs
e.g. 0x11 —> 0x01

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```

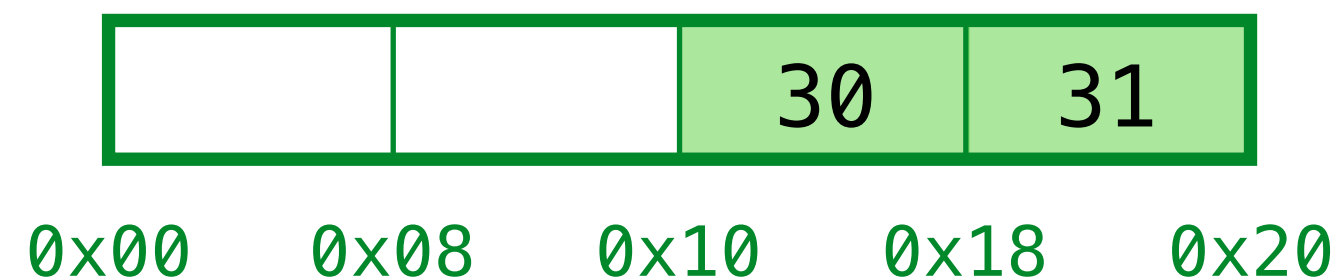
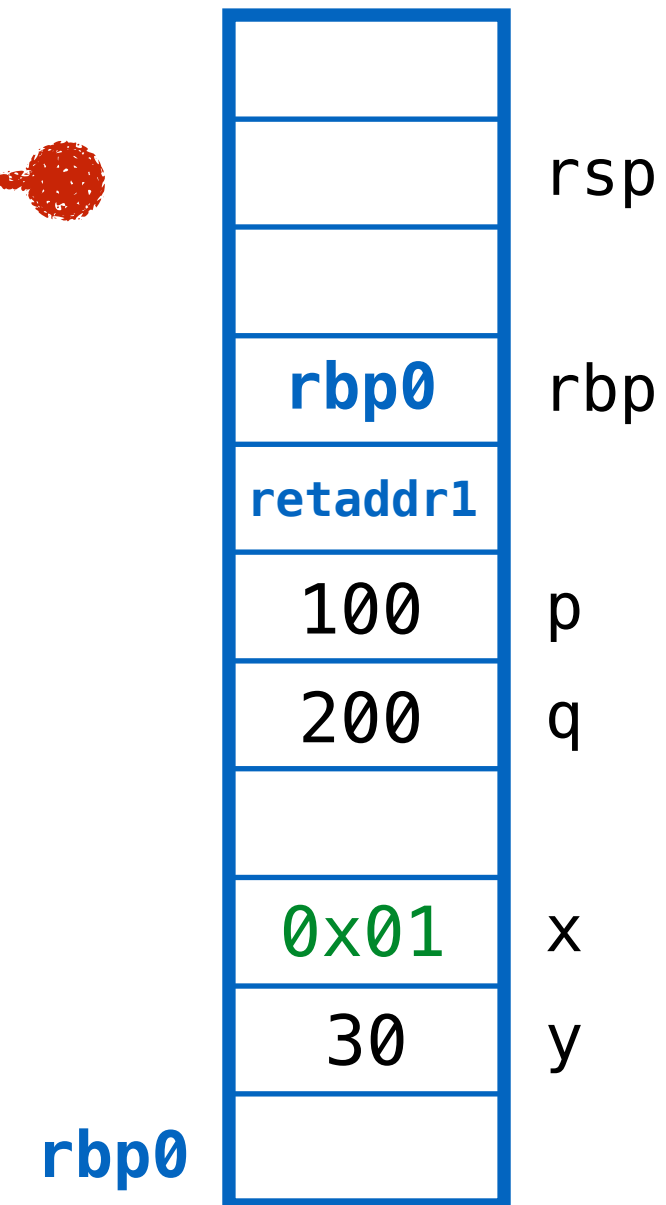


1. Compute **FORWARD** addrs
e.g. 0x11 —> 0x01

r15 2. **REDIRECT** addrs on stack

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
  let tmp = (p, q)  
  in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
  , x = (y, y + 1)  
  , z = foo(100, 200)  
in  
  x[0] + z
```

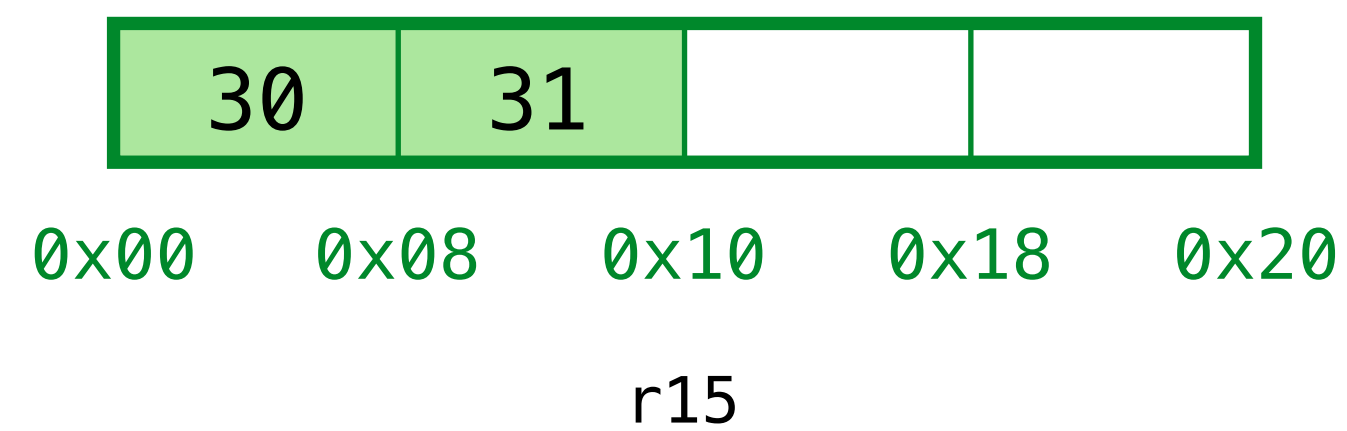
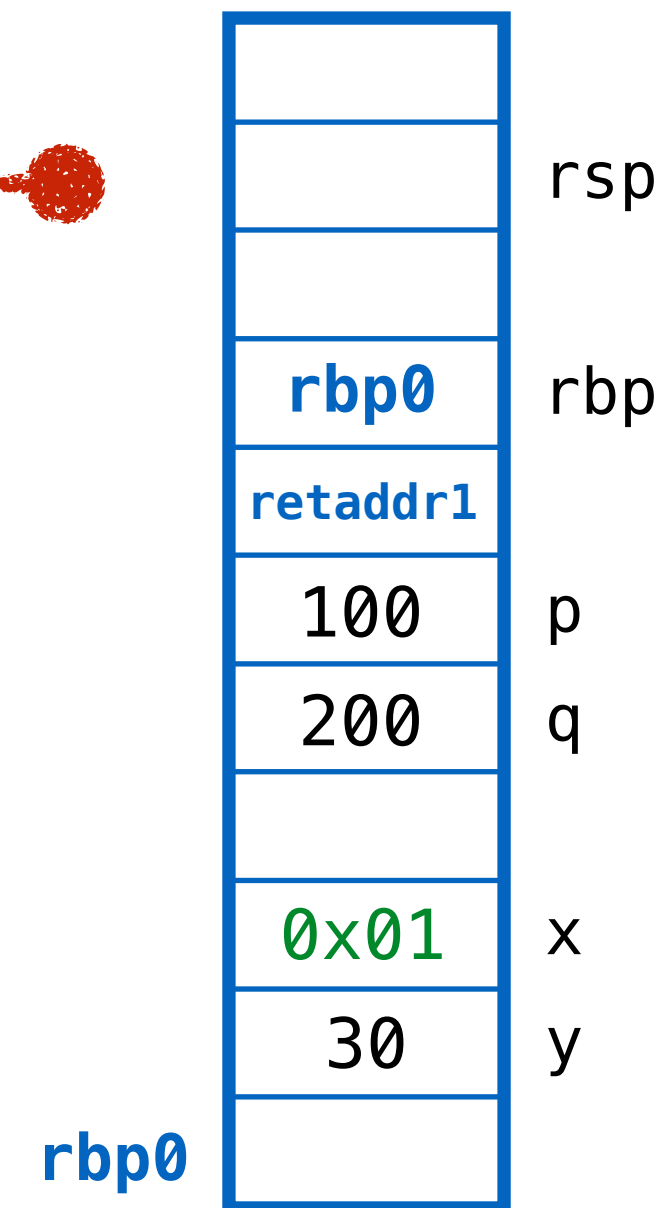


r15

1. Compute **FORWARD** addrs
e.g. 0x11 → 0x01
2. **REDIRECT** addrs on stack
3. **COMPACT** cells on heap

ex3: garbage in the middle (with stack)

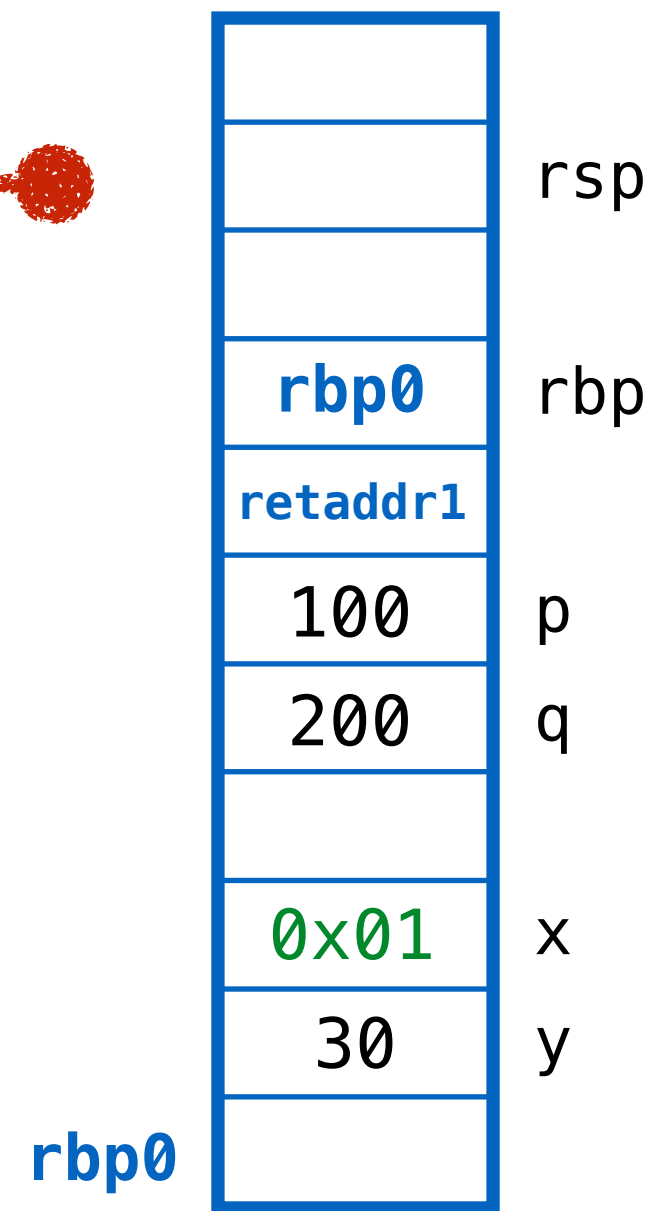
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



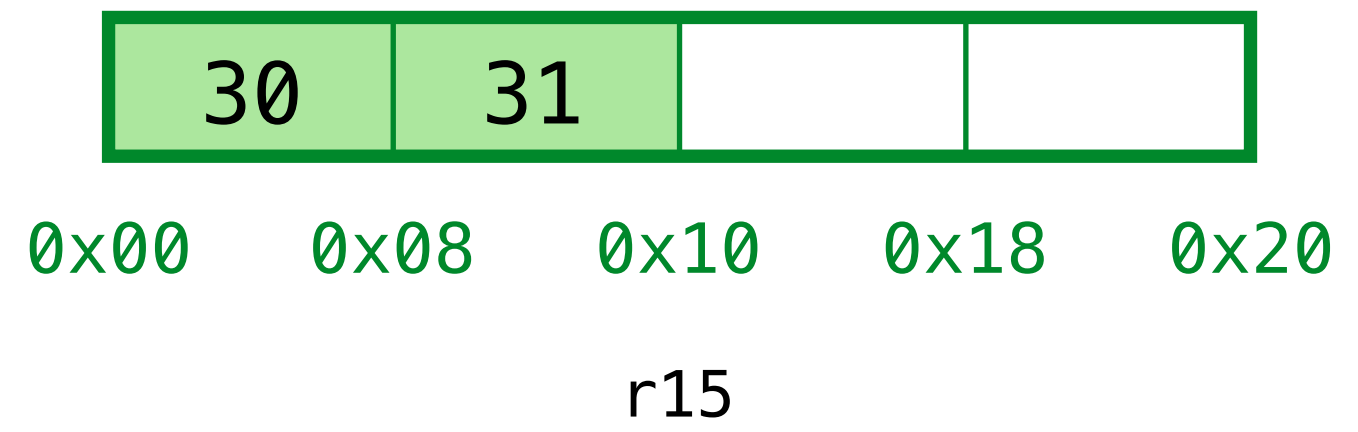
1. Compute **FORWARD** addrs
e.g. **0x11** —> **0x01**
2. **REDIRECT** addrs on stack
3. **COMPACT** cells on heap

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```

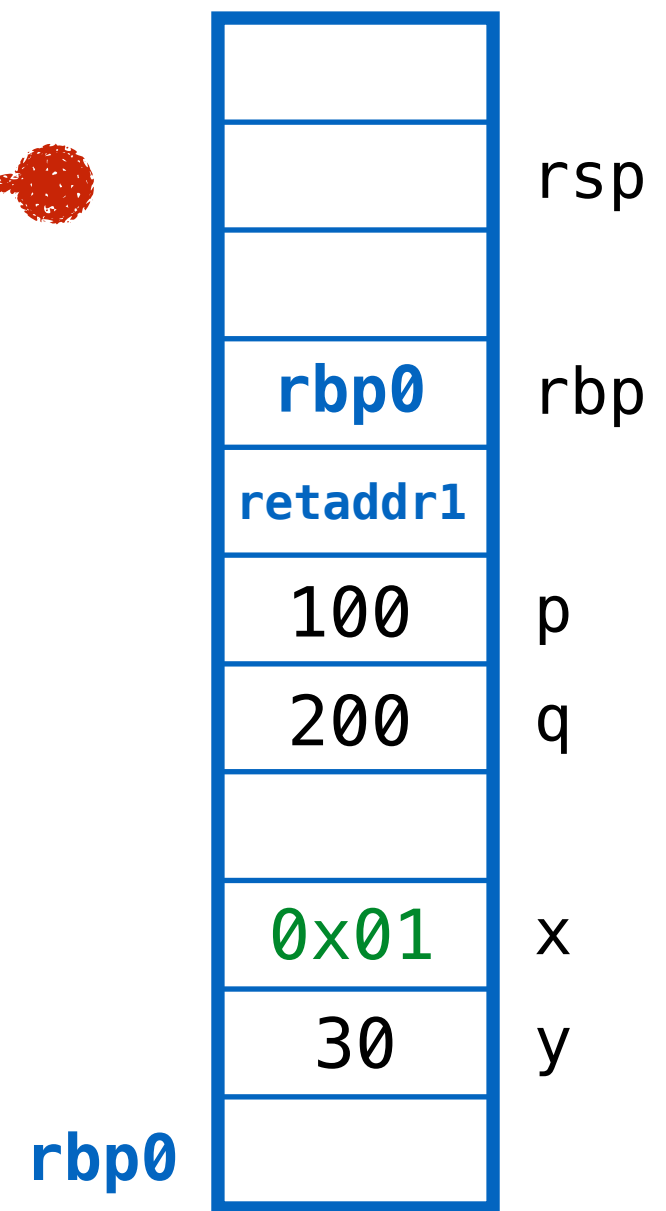


Yay! Have space for (p, q)

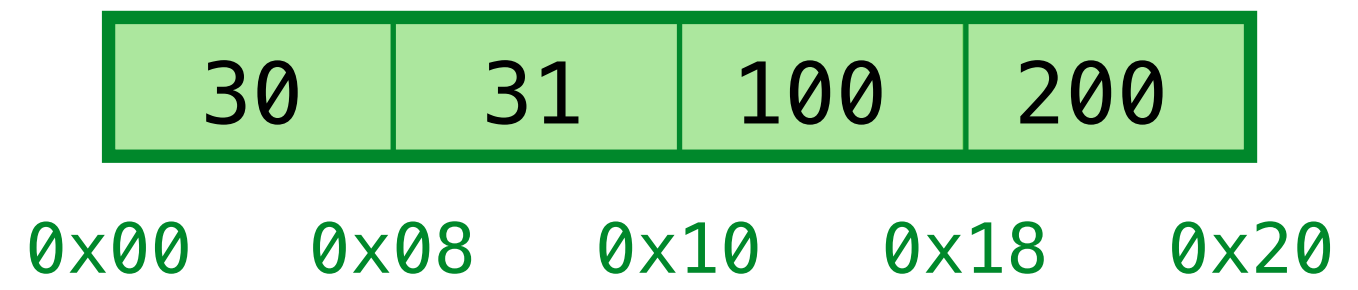


ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```

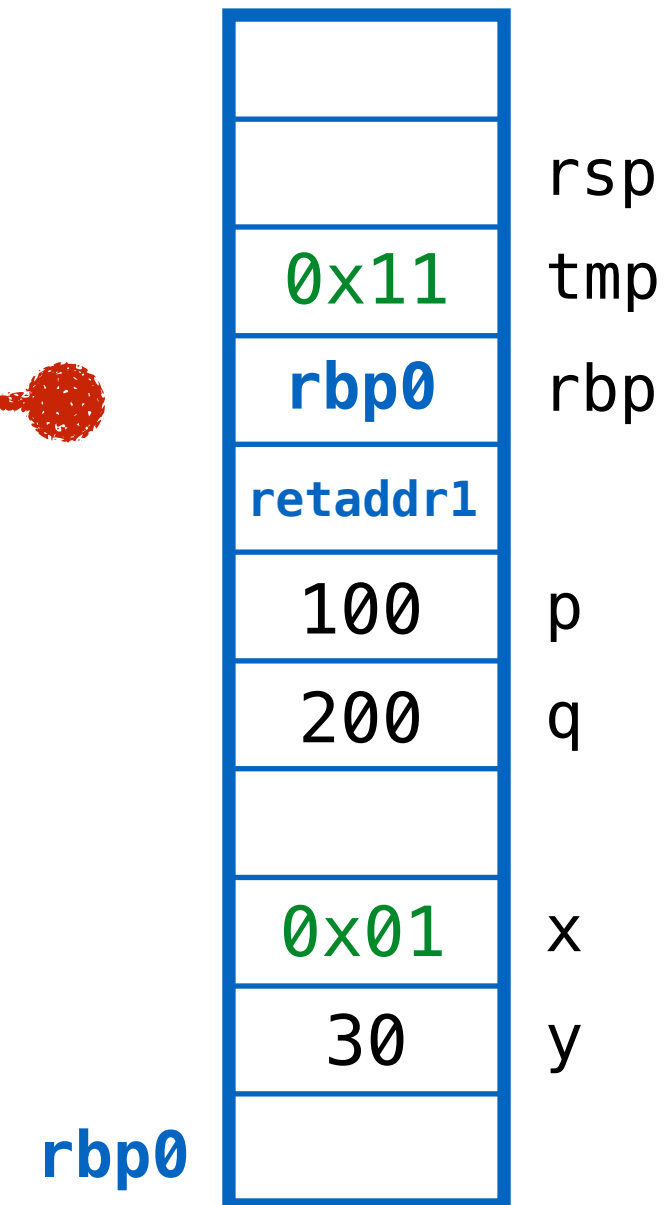


Yay! Have space for (p, q)

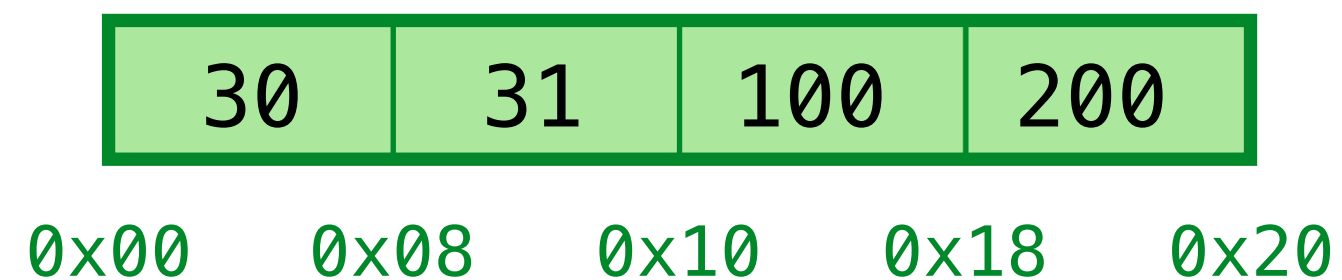


ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



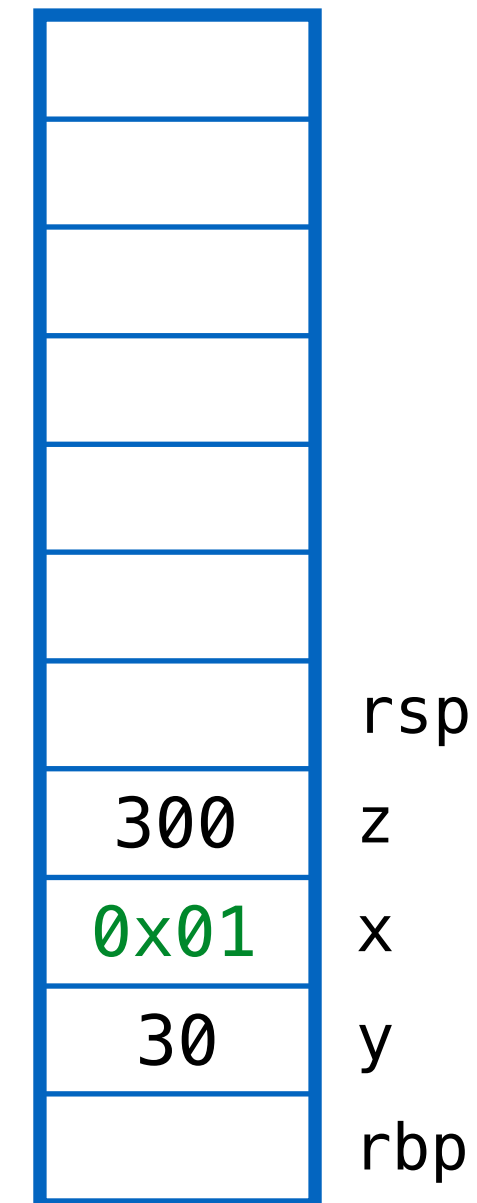
Return (rax) = 300



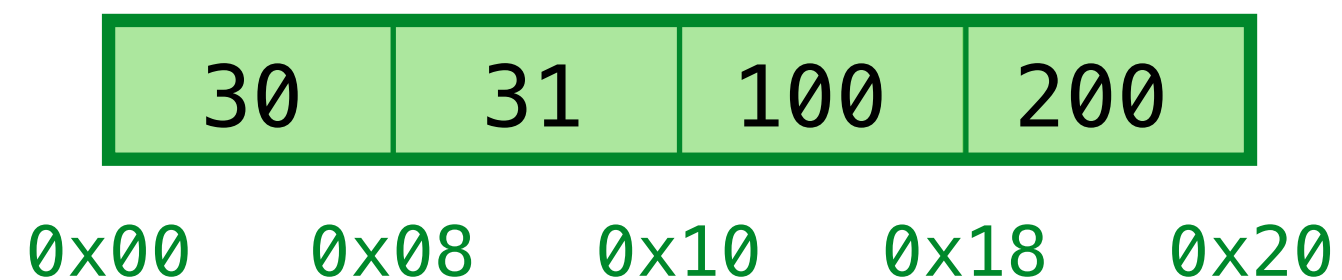
r15

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
    x[0] + z
```



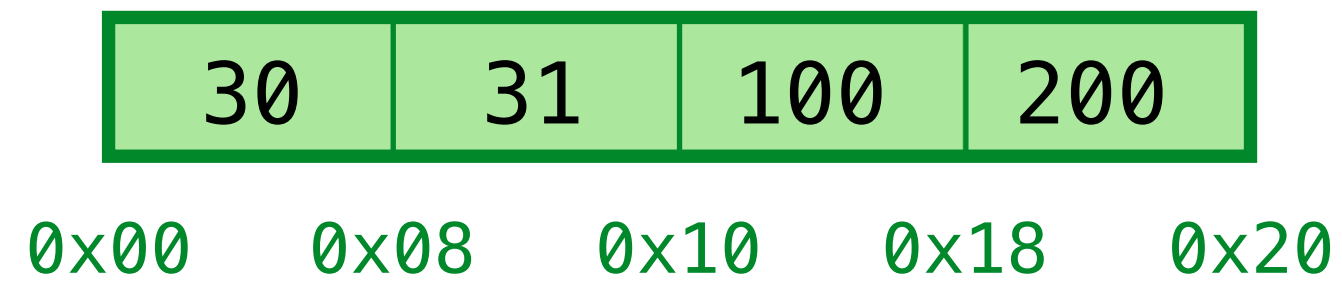
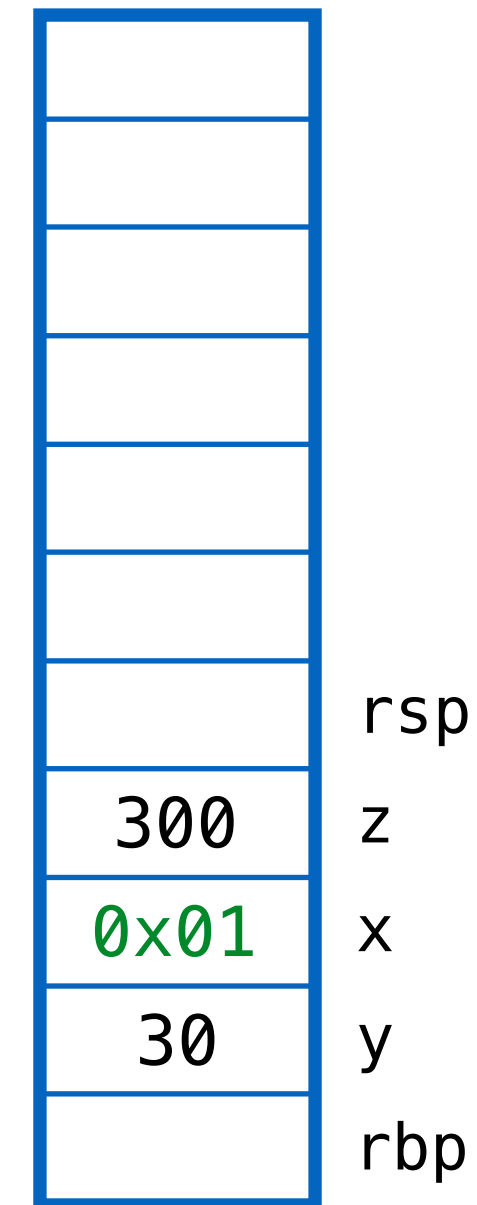
Return (rax) = 300



r15

ex3: garbage in the middle (with stack)

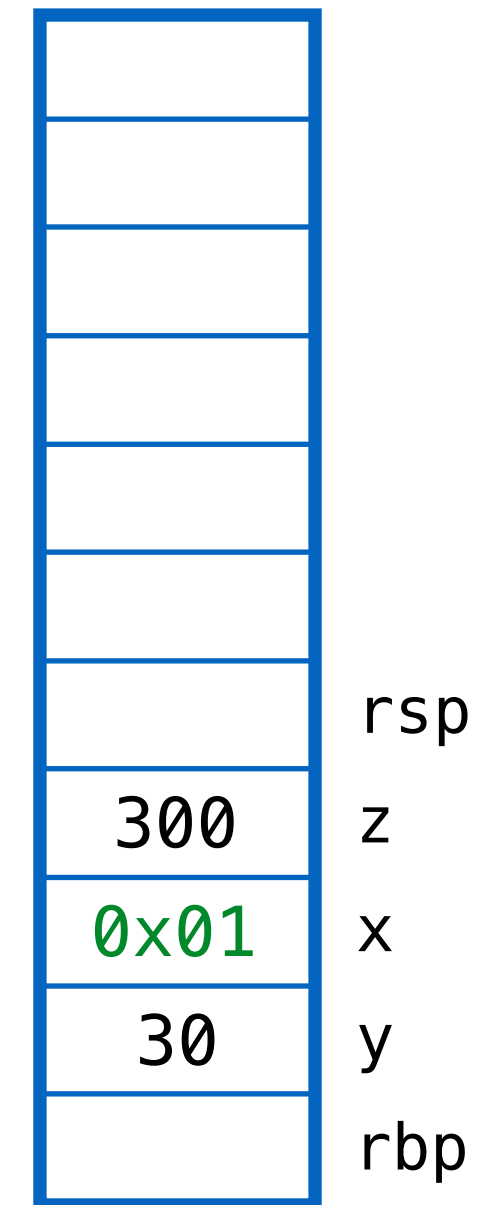
```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
x[0] + z
```



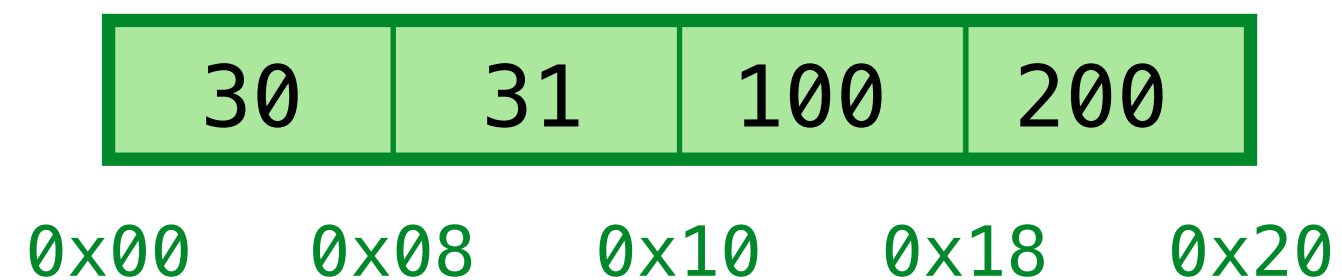
r15

ex3: garbage in the middle (with stack)

```
def foo(p, q):  
    let tmp = (p, q)  
    in tmp[0] + tmp[1]  
  
let y = foo(10, 20)  
    , x = (y, y + 1)  
    , z = foo(100, 200)  
in  
x[0] + z
```



Return (rax) = 30+300 = 330



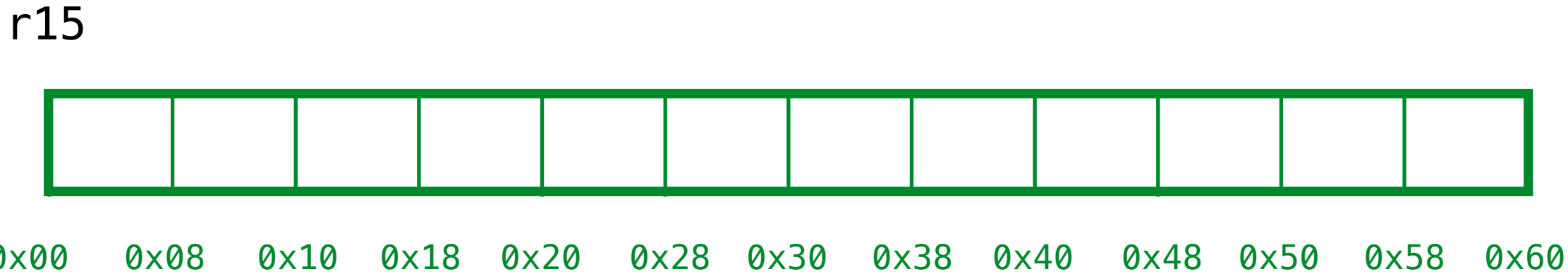
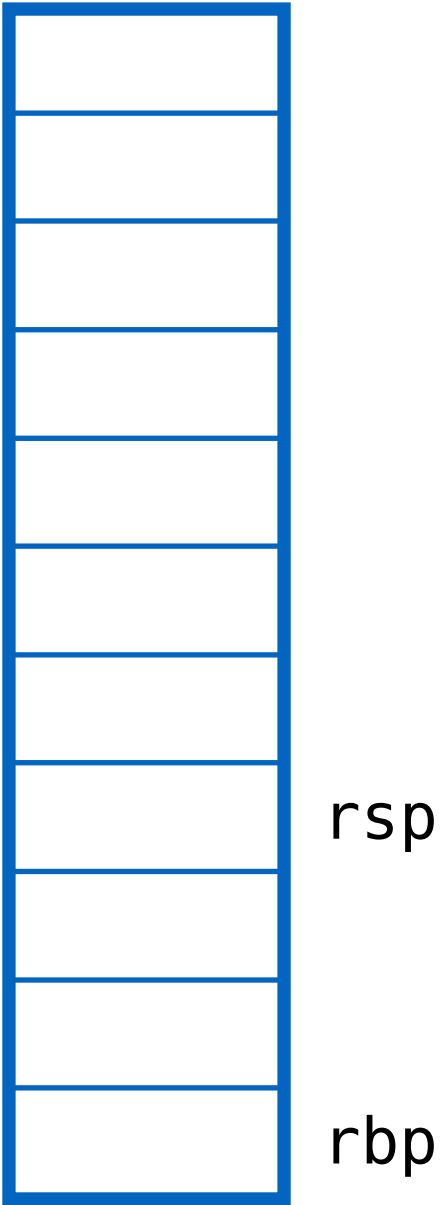
r15

Garter / GC

Example 4

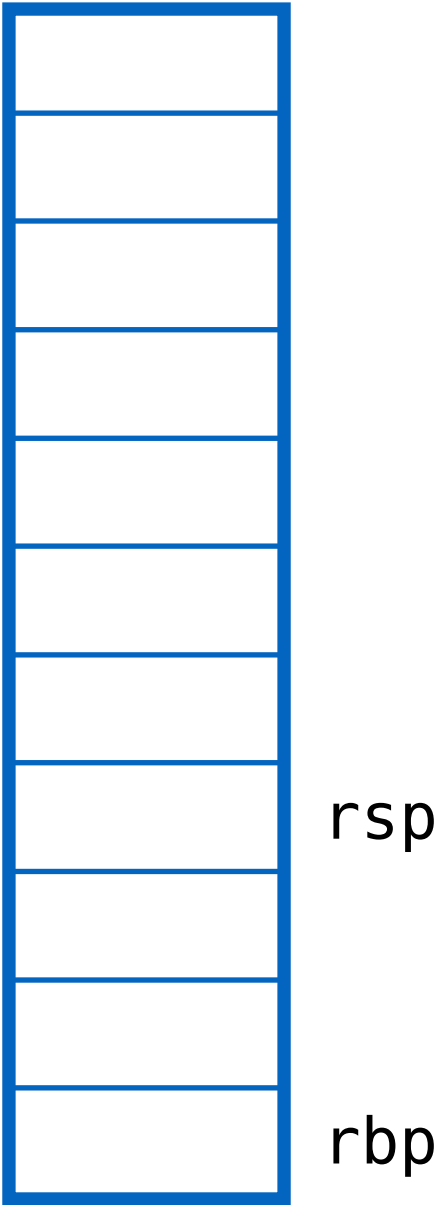
ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i, range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
    , l = range(t1, t1 + 3)  
in  
(1000, l)
```

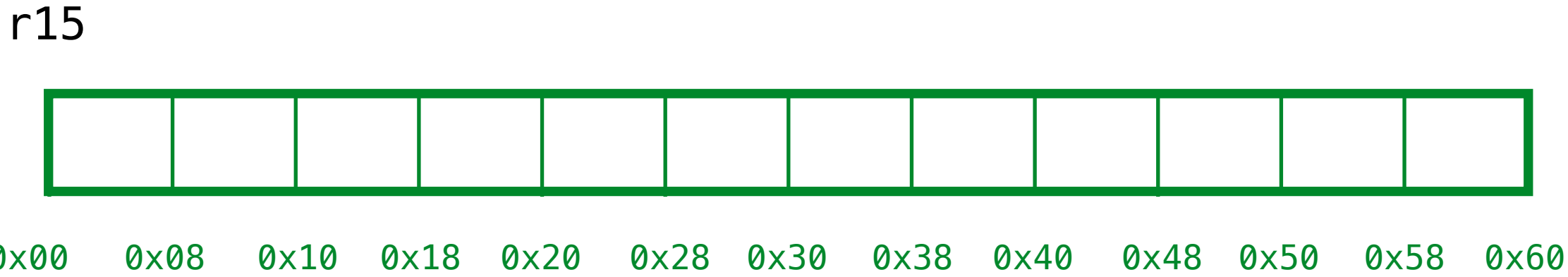


ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i, range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
    , l = range(t1, t1 + 3)  
in  
(1000, l)
```



call range(0, 3)



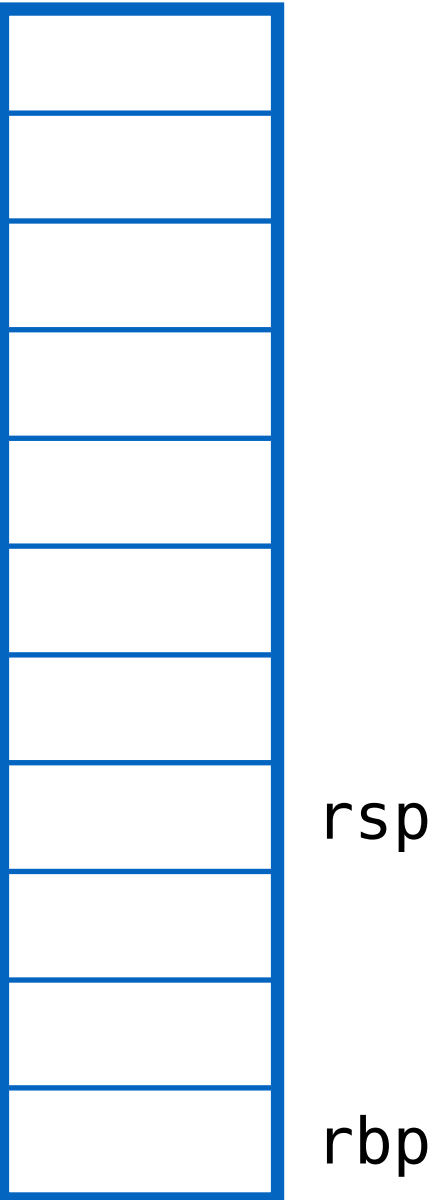
ex4: recursive data

```

def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

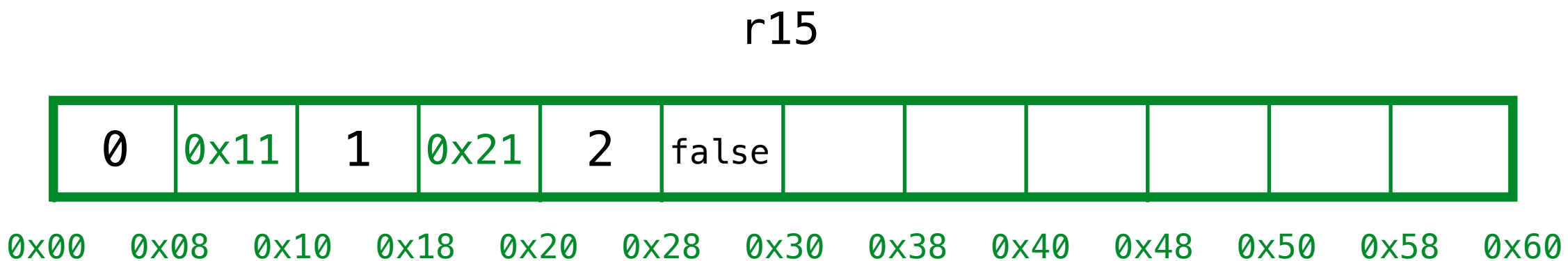
def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
    
```

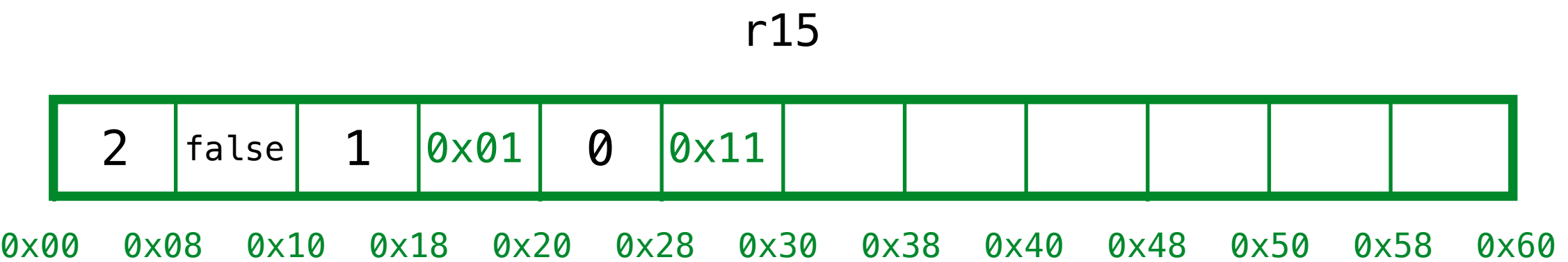


QUIZ: What is heap when range(0, 3) returns?

(A)

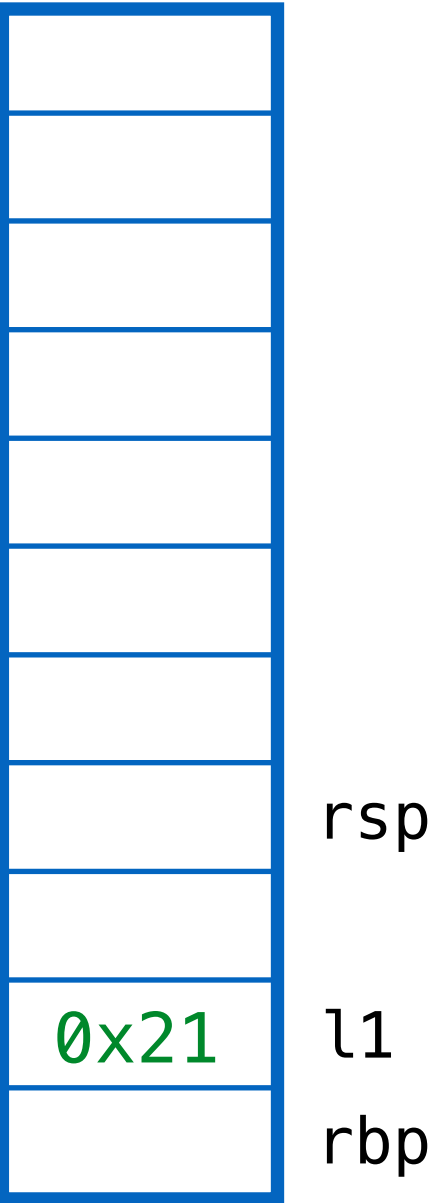


(B)



ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i, range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



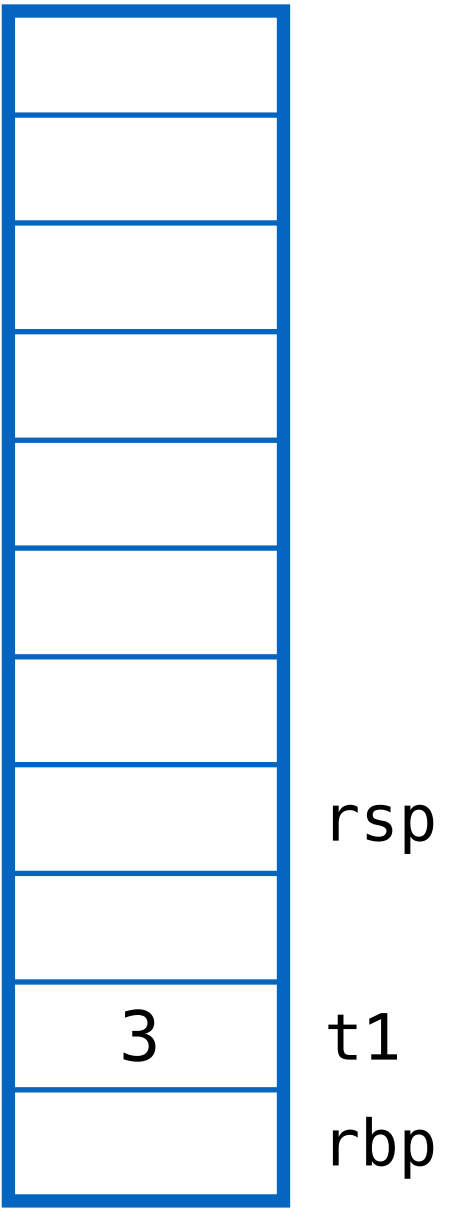
r15



0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

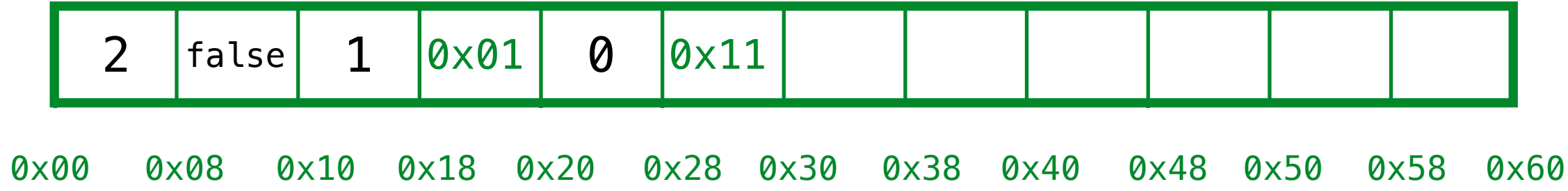
ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i, range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



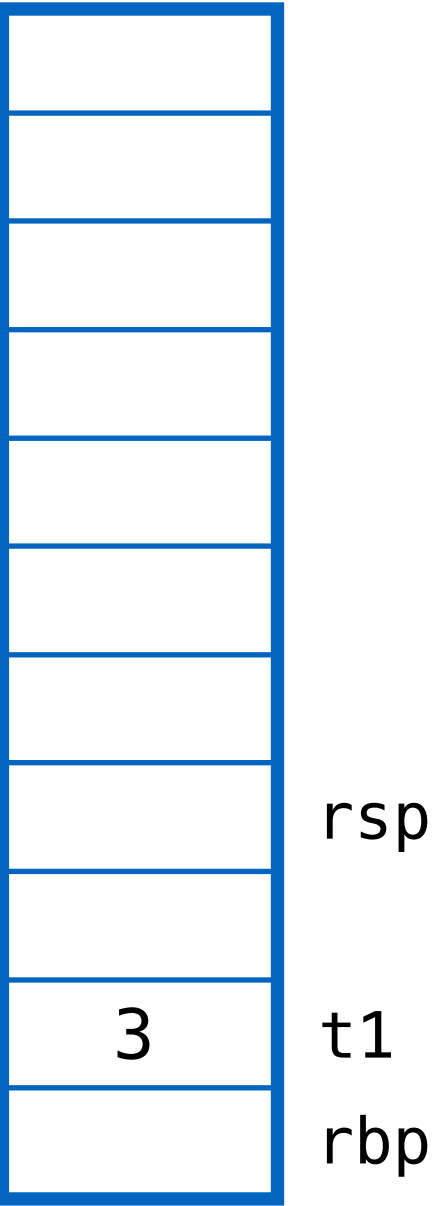
Result sum(0x11) = 3

r15



ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i, range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, = range(t1, t1 + 3)  
in  
(1000, l)
```



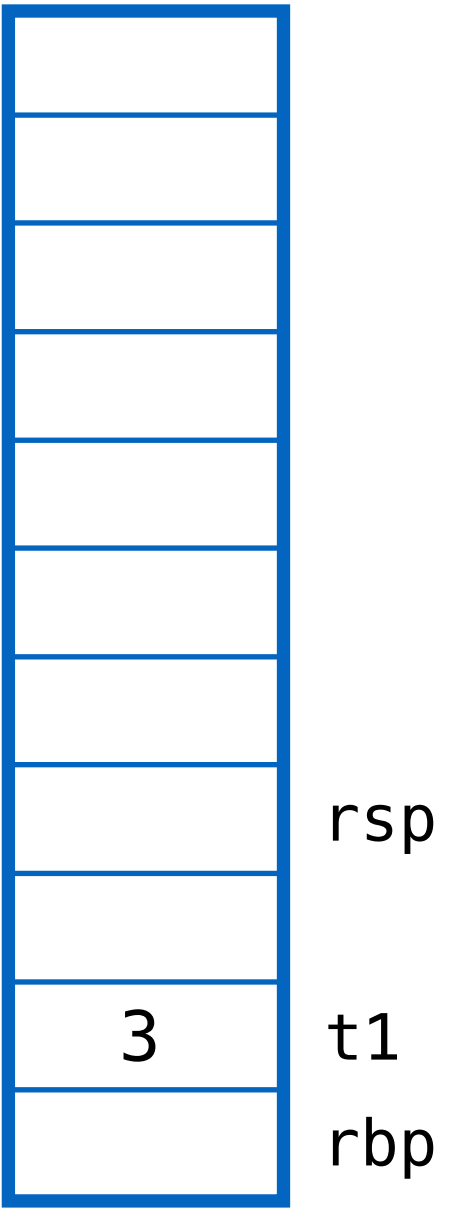
r15



0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

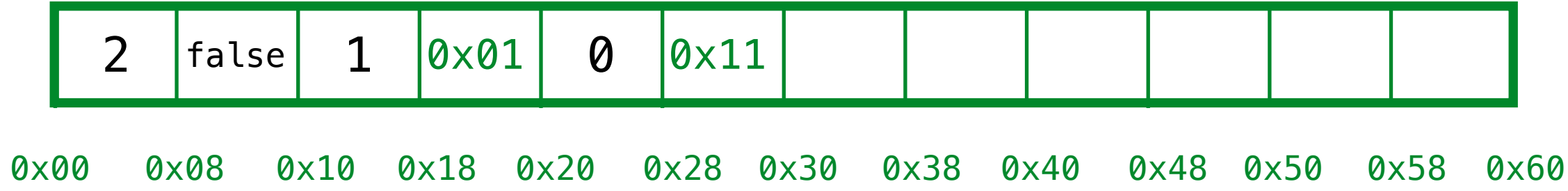
ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i, range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



call range(3,6)

r15



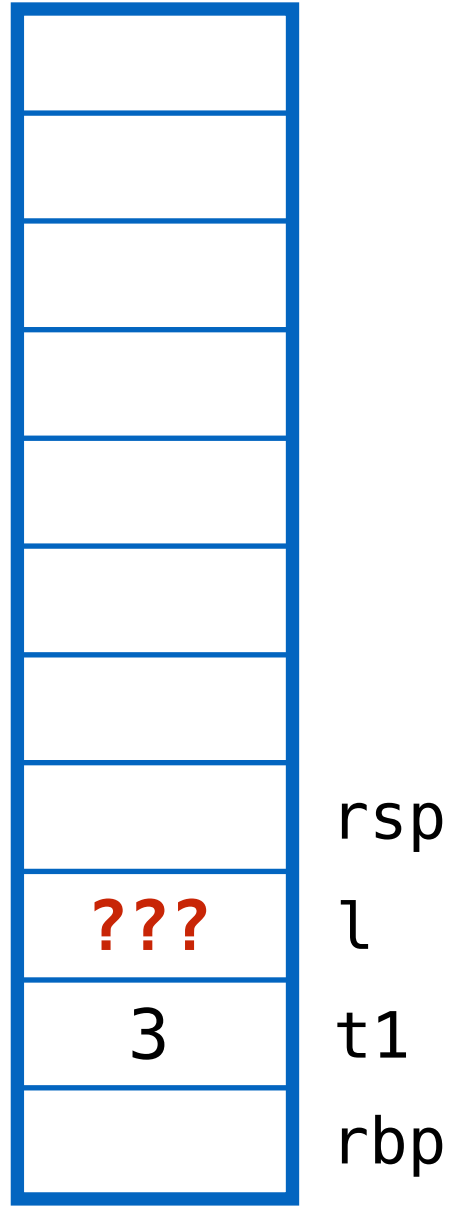
ex4: recursive data

```

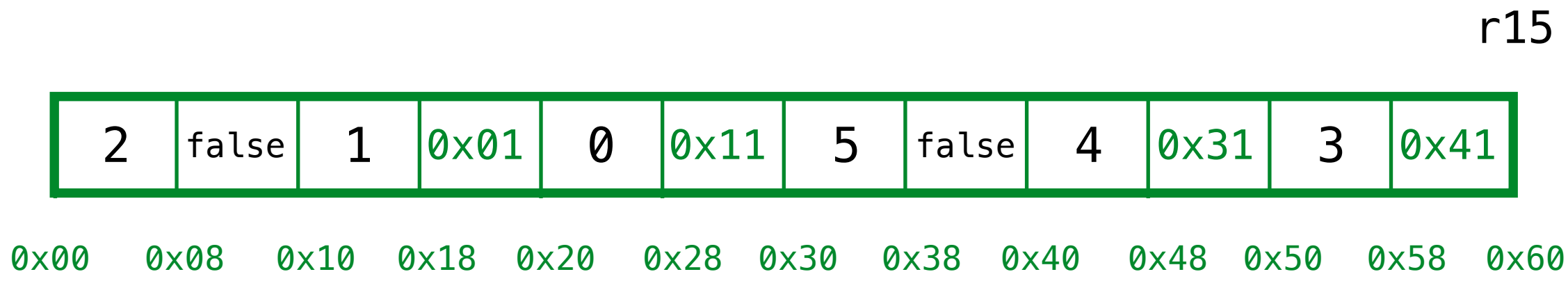
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
    
```



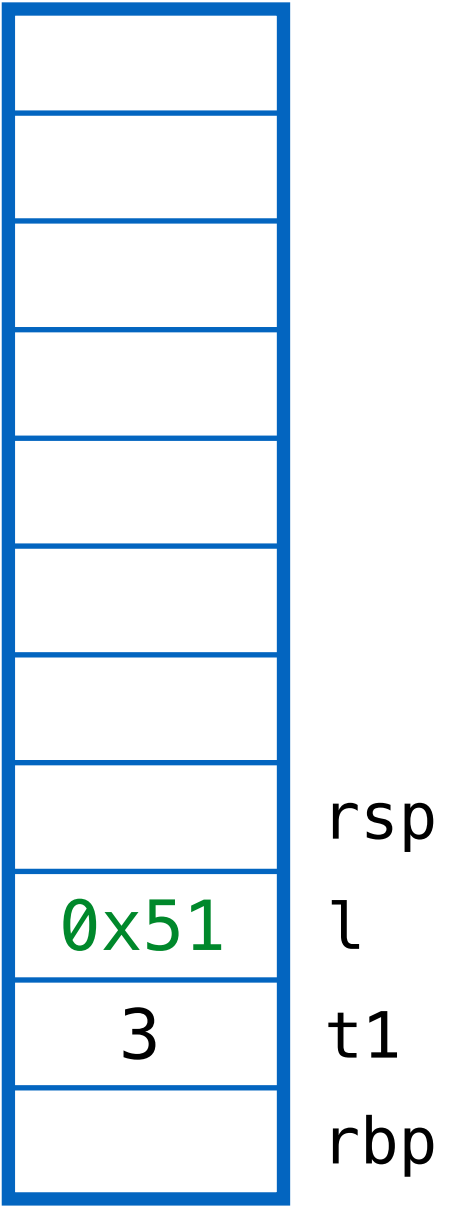
call range(3,6)



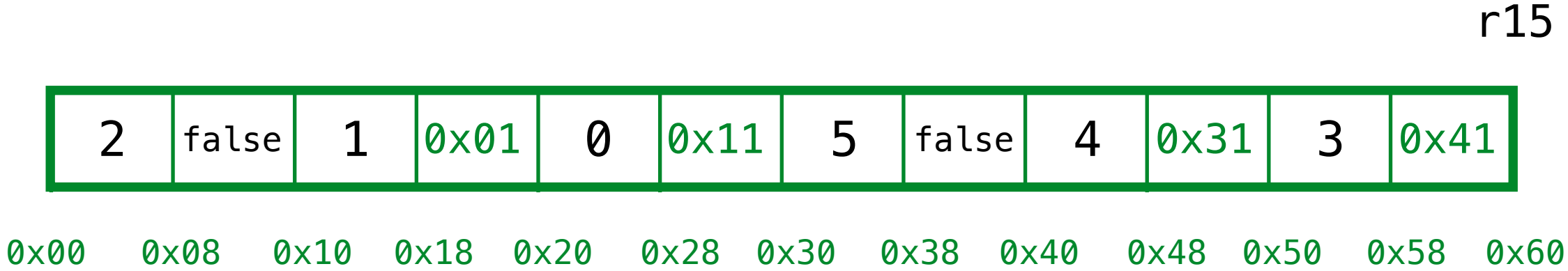
QUIZ: What is the value of **l?**
(A) 0x30 (B) 0x31 (C) 0x50 (D) 0x51 (E) 0x60

ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i, range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



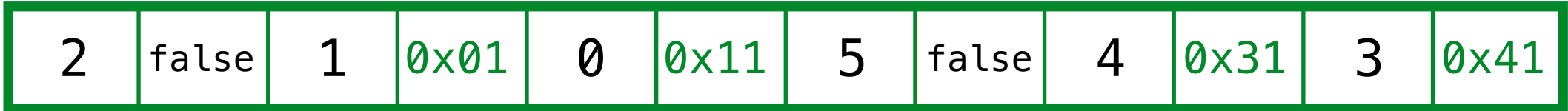
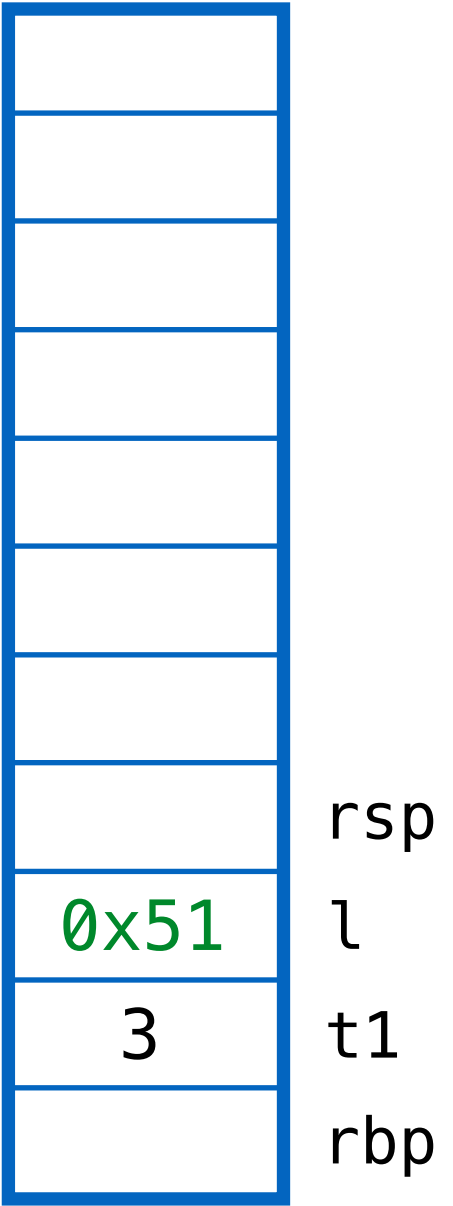
Yikes! Out of Memory!



ex4: recursive data

QUIZ: Which cells are "live" on the heap?

- (A) 0x00
- (B) 0x10
- (C) 0x20
- (D) 0x30
- (E) 0x40
- (F) 0x50



0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

r15

ex4: recursive data

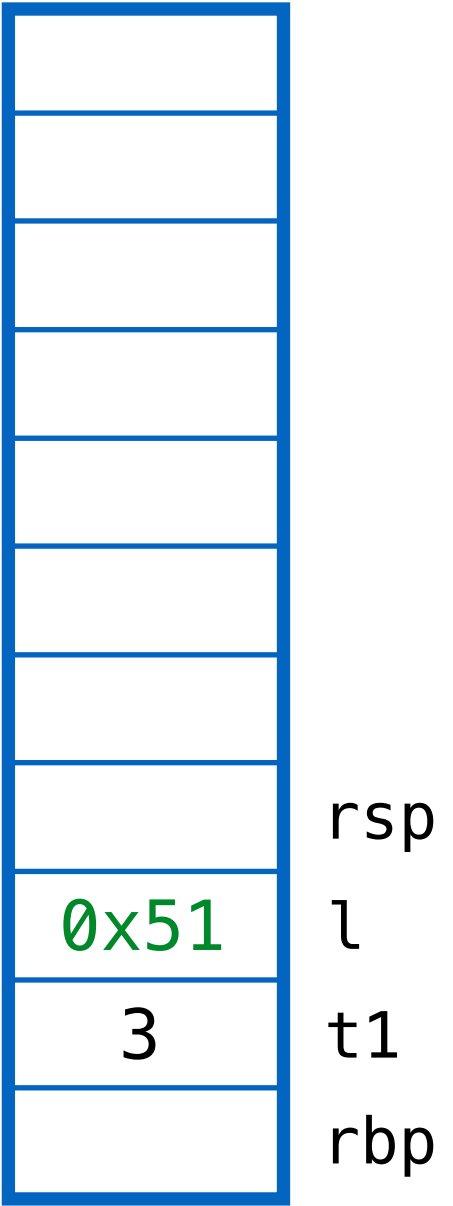
```

def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)

```



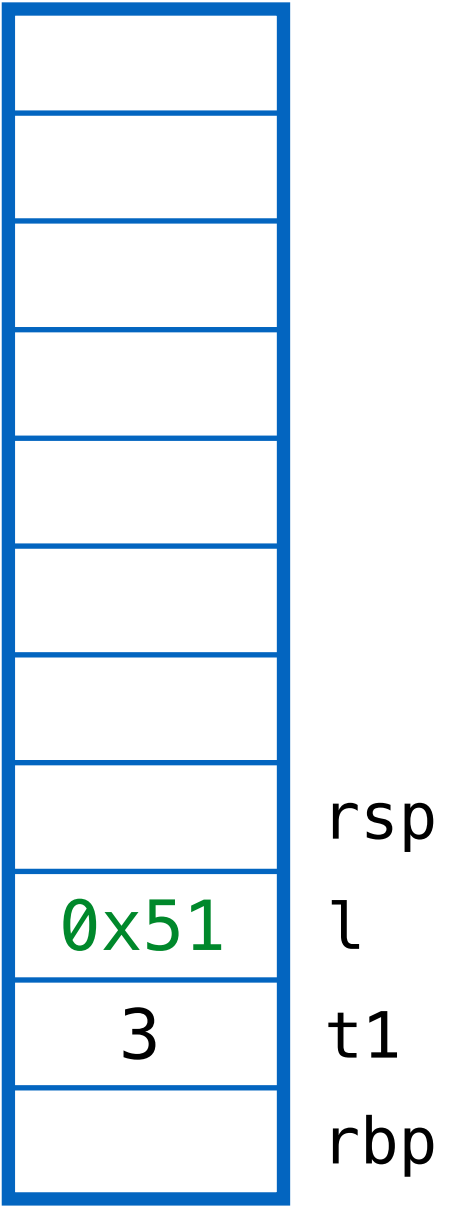
2	false	1	0x01	0	0x11	5	false	4	0x31	3	0x41
---	-------	---	------	---	------	---	-------	---	------	---	------

0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

1. MARK live addrs
2. Compute FORWARD addrs
3. REDIRECT addrs on stack
4. COMPACT cells on heap

ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



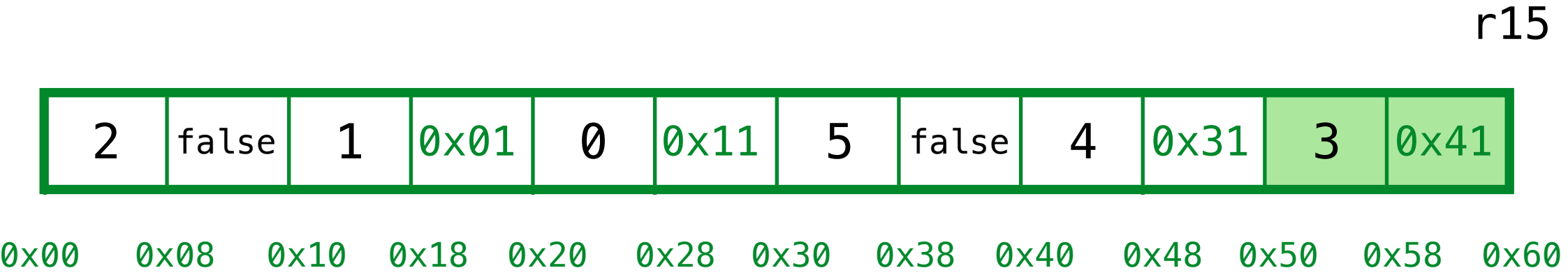
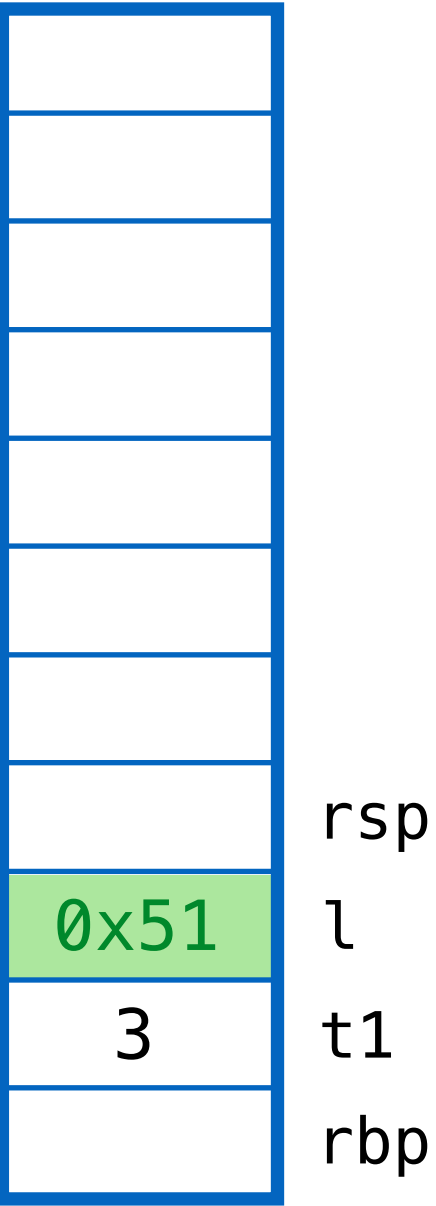
2	false	1	0x01	0	0x11	5	false	4	0x31	3	0x41
---	-------	---	------	---	------	---	-------	---	------	---	------

0x00 0x08 0x10 0x18 0x20 0x28 0x30 0x38 0x40 0x48 0x50 0x58 0x60

1. MARK live addrs
reachable from stack

ex4: recursive data

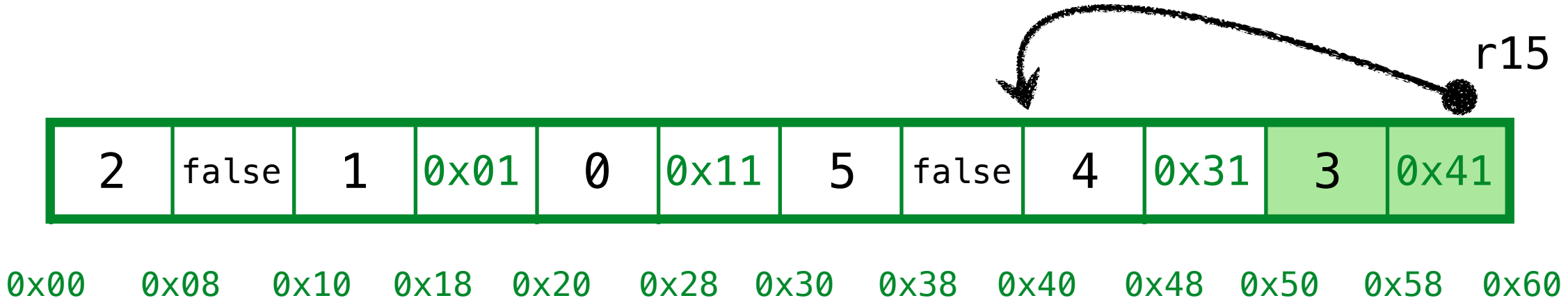
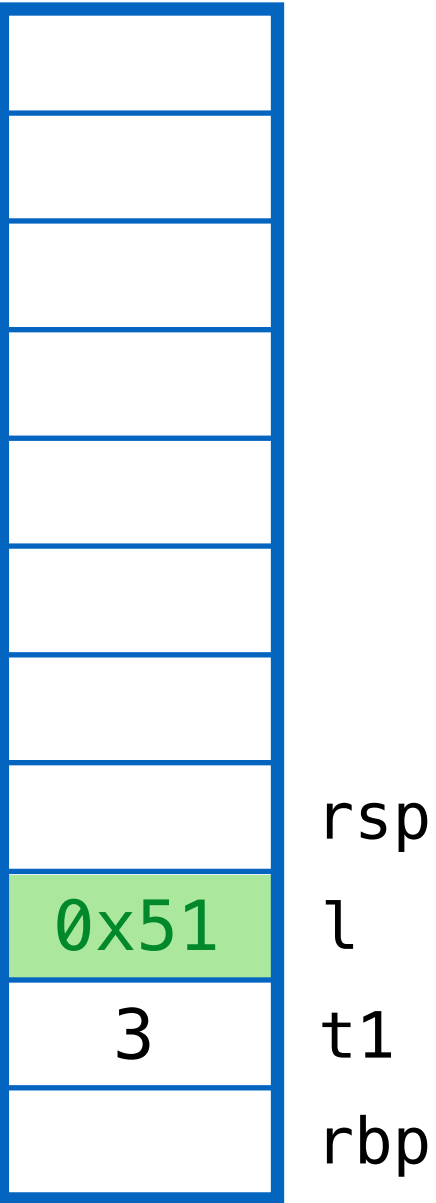
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



1. MARK live addrs
reachable from stack

ex4: recursive data

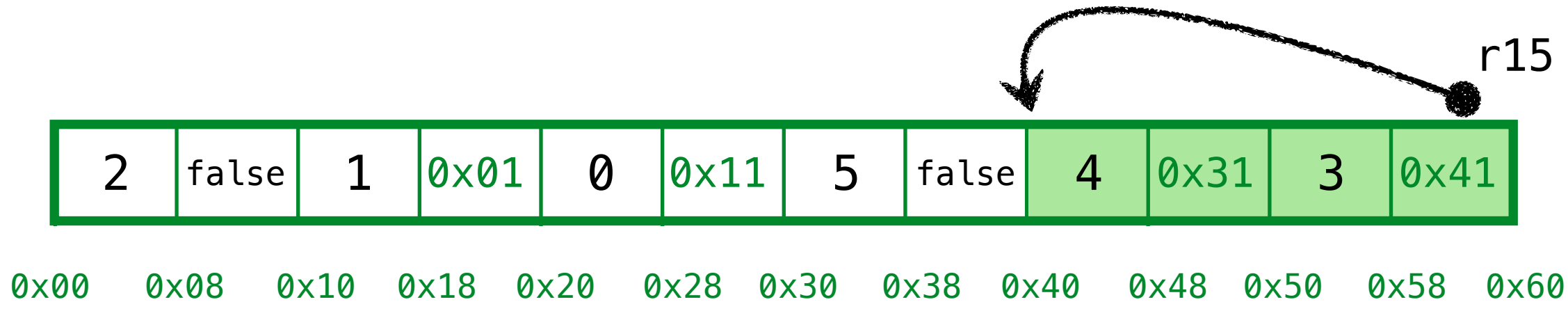
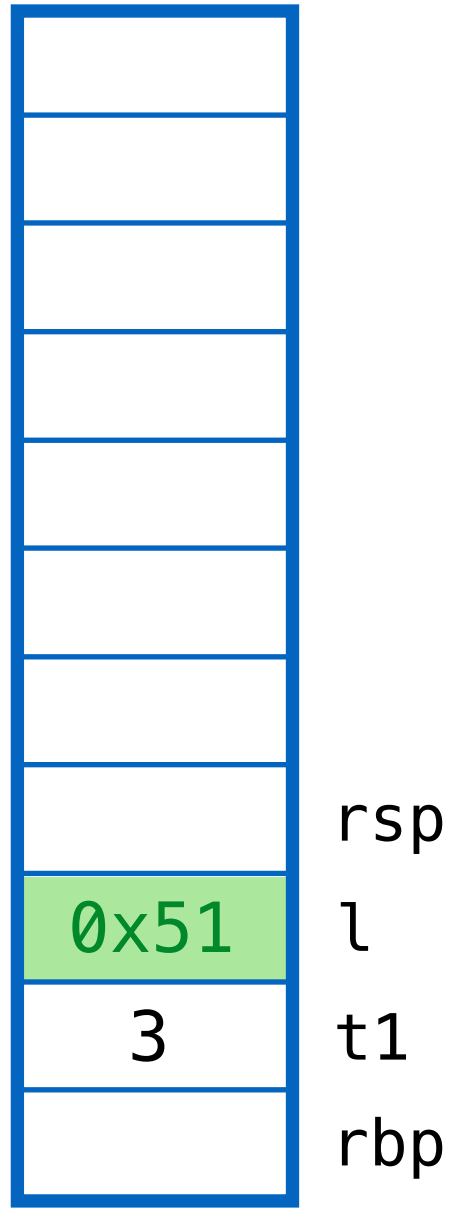
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



1. MARK live addrs
reachable from stack

ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
    , l = range(t1, t1 + 3)  
in  
(1000, l)
```



1. MARK live addrs
reachable from stack

ex4: recursive data

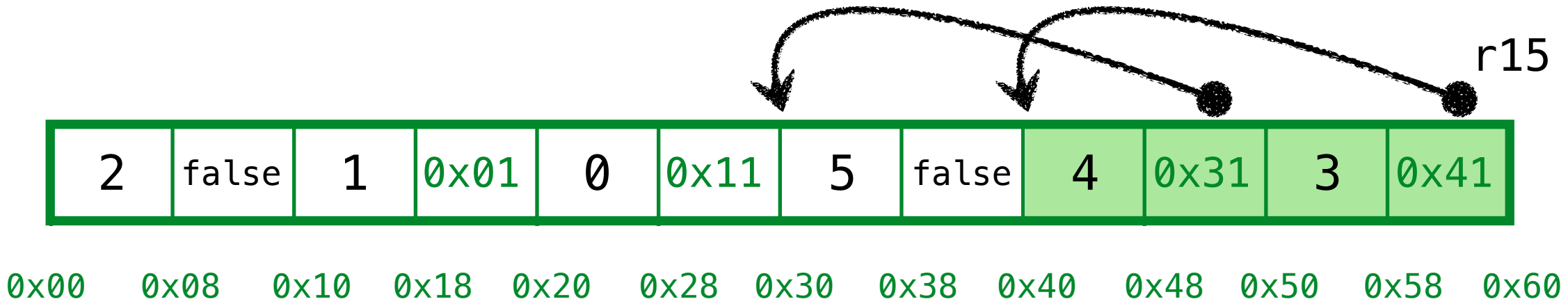
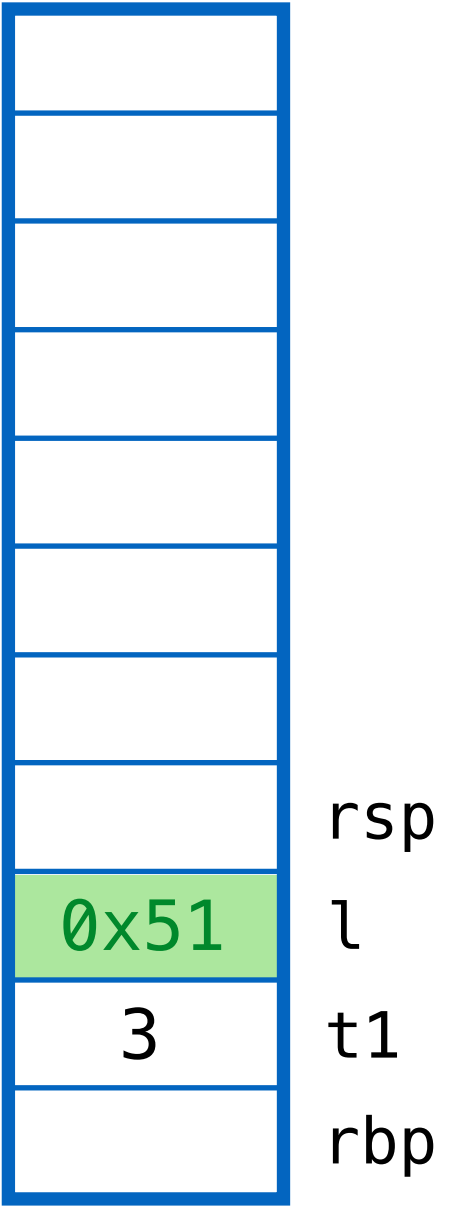
```

def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)

```



1. MARK live addrs
reachable from stack

ex4: recursive data

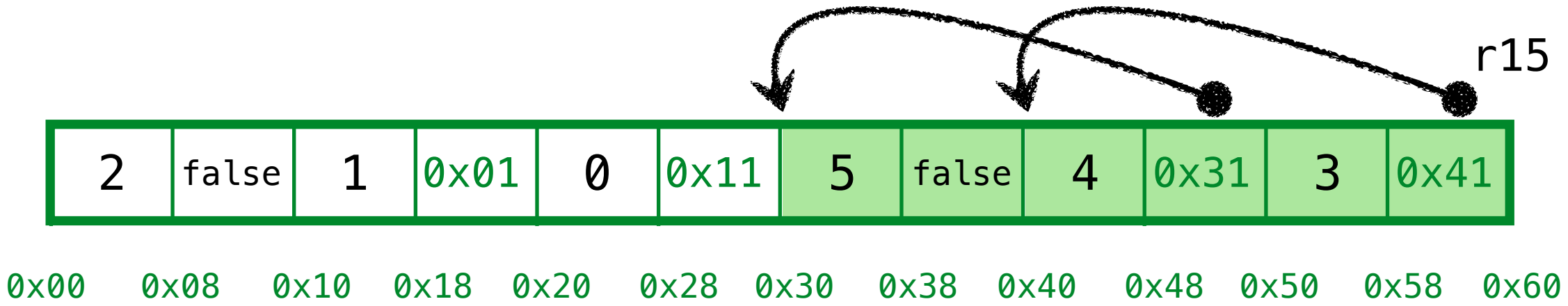
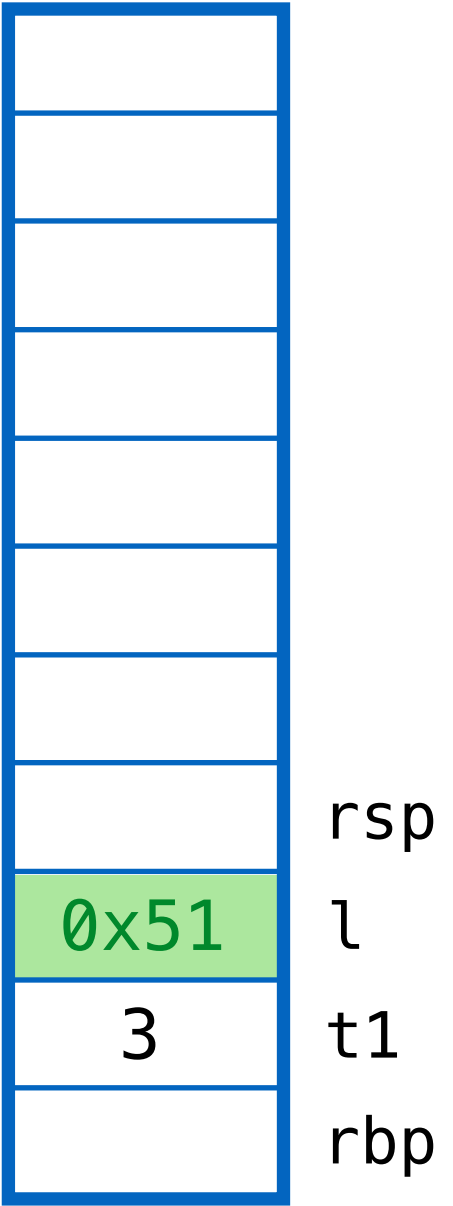
```

def range(i, j):
    if (j <= i): false else: (i, range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)

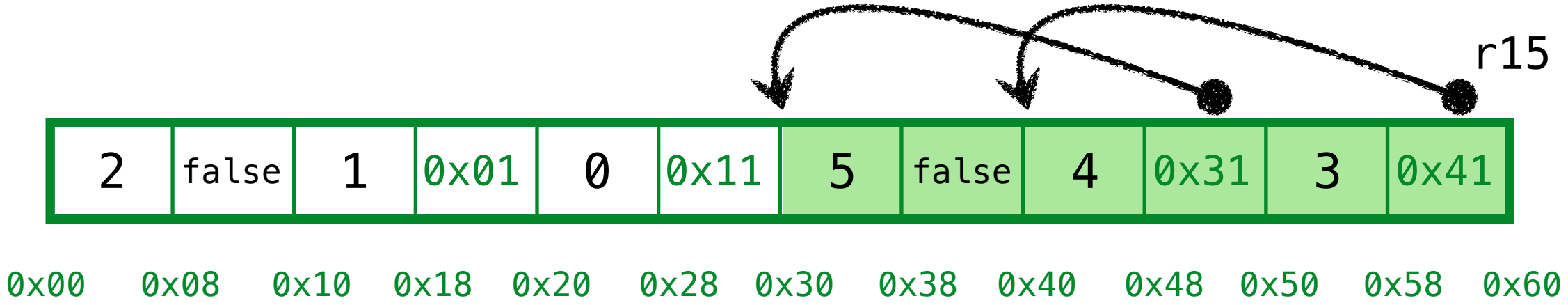
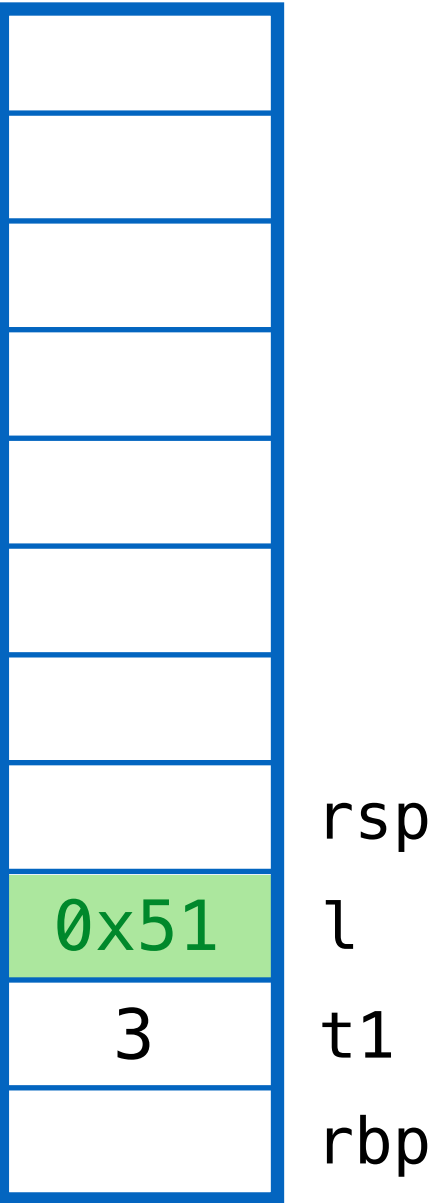
```



1. MARK live addrs
 reachable from stack

ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



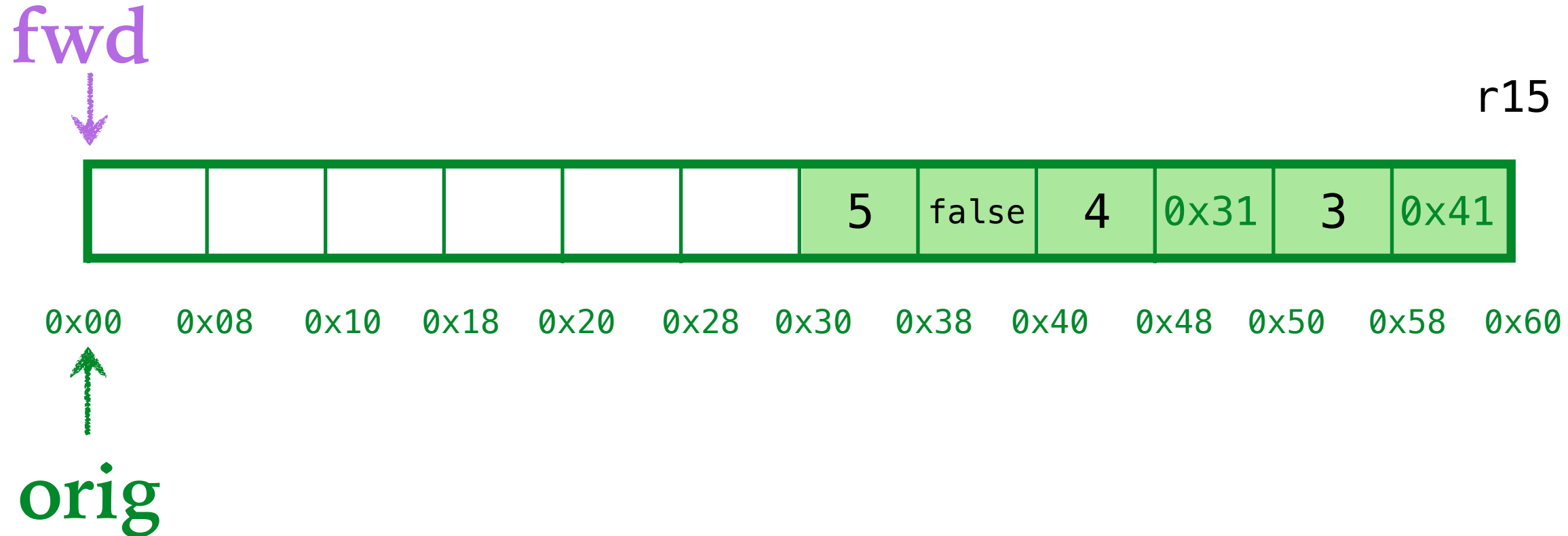
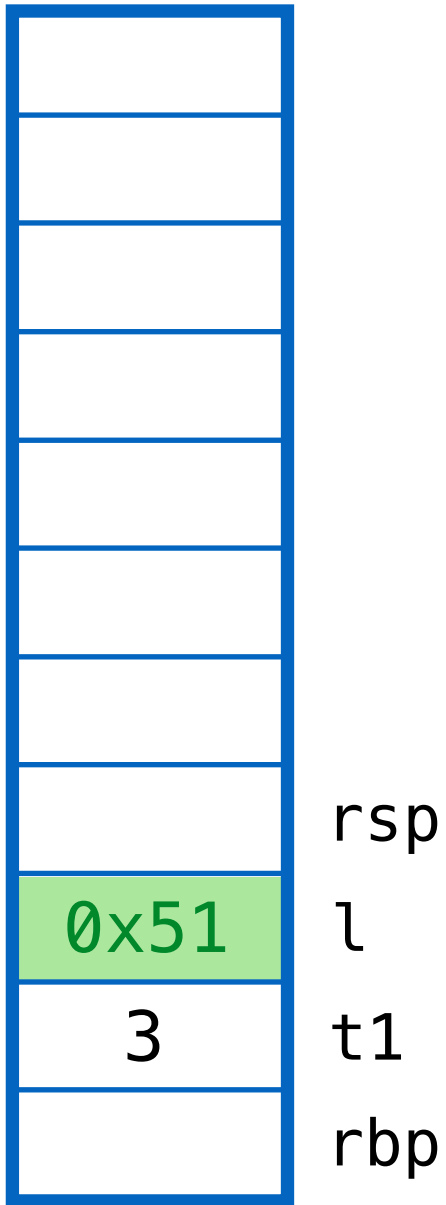
Done!

ex4: recursive data

```
def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

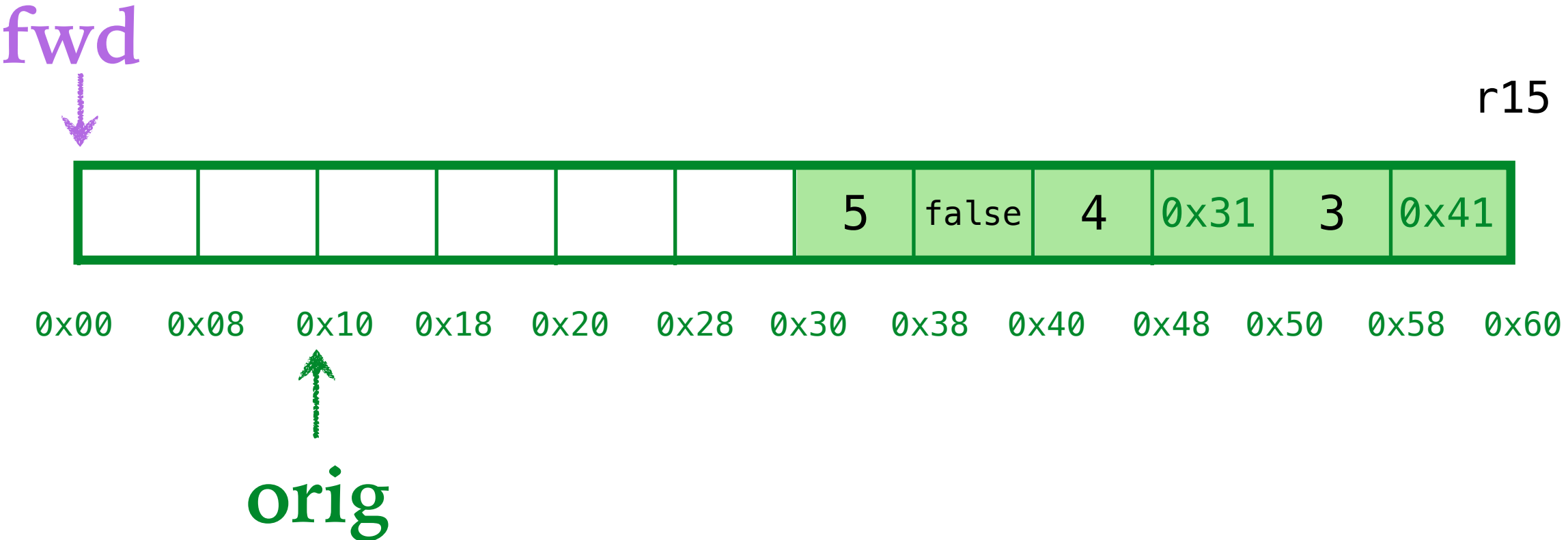
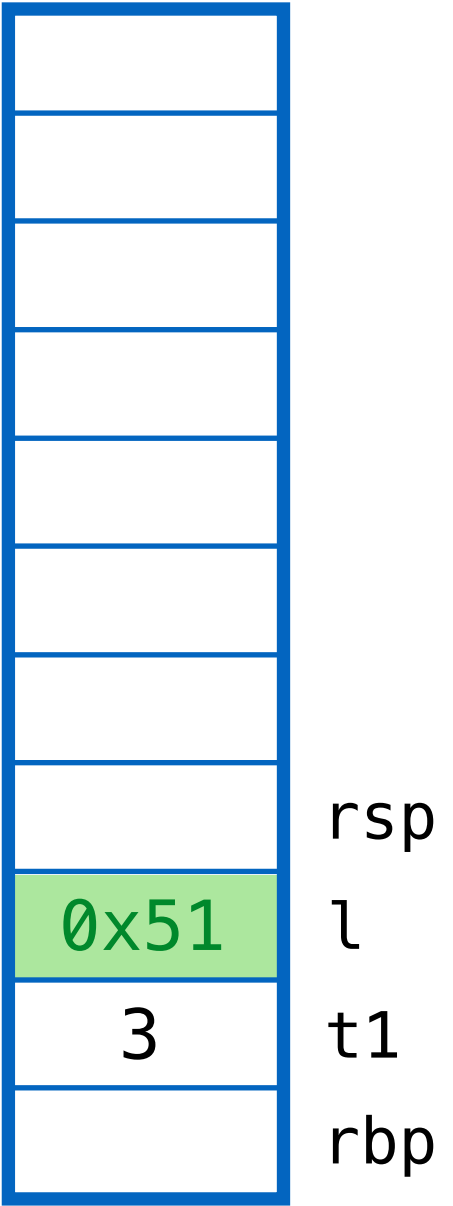
let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



2. Compute **FORWARD** addrs

ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i, range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
  , l = range(t1, t1 + 3)  
in  
(1000, l)
```



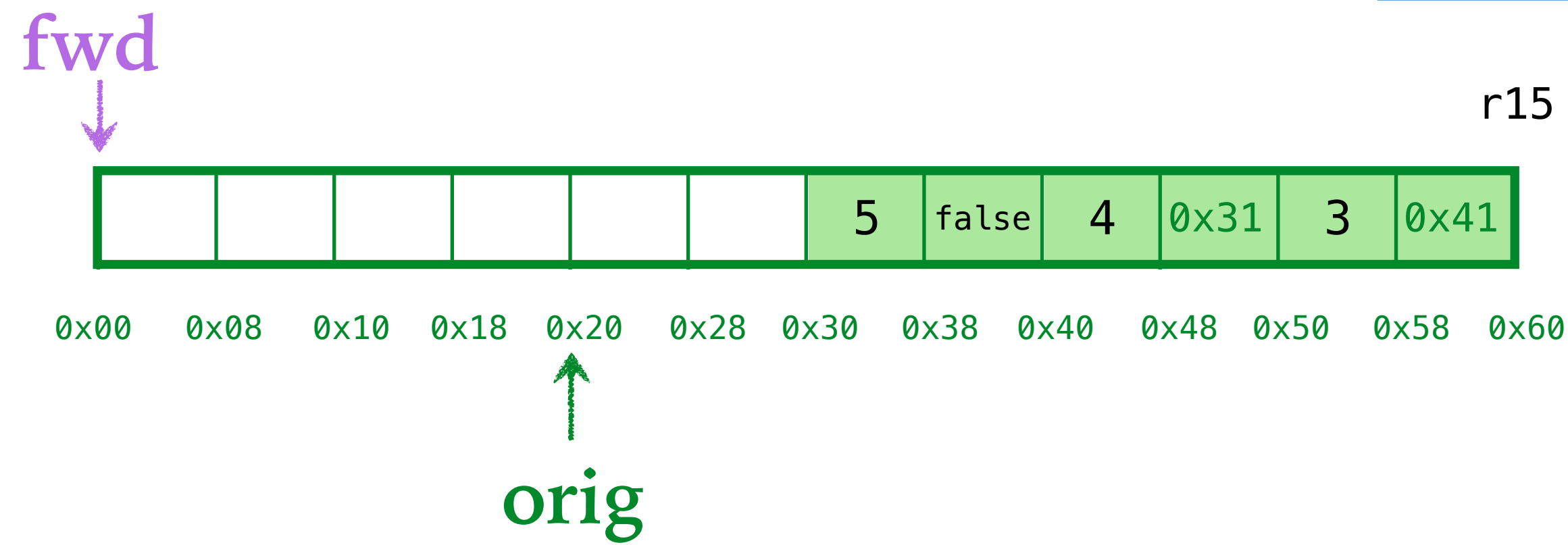
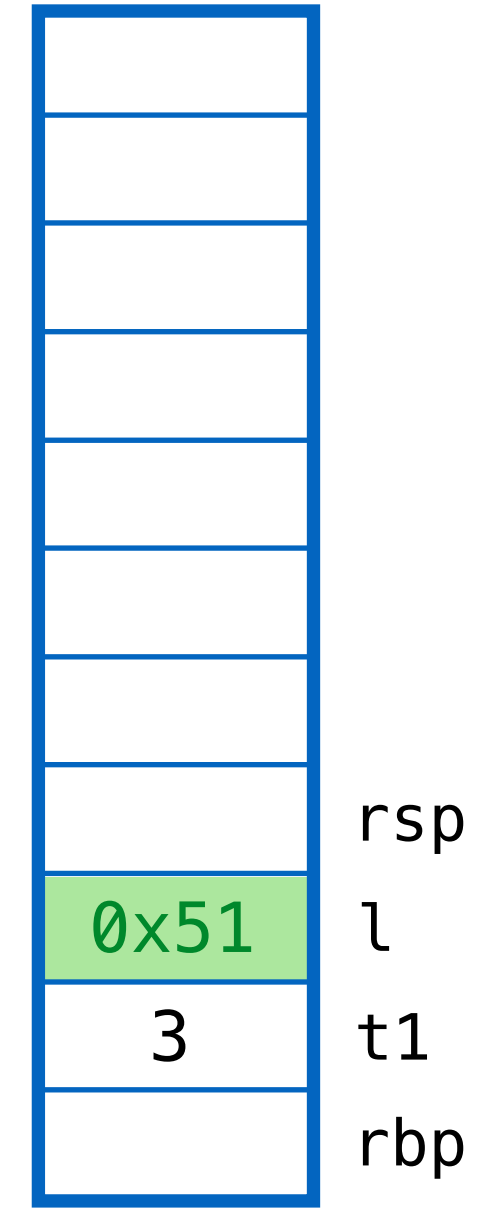
2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):
  if (j <= i): false else: (i,range(i+1, j))

def sum(l):
  if l == false: 0 else: l[0] + sum(l[1])

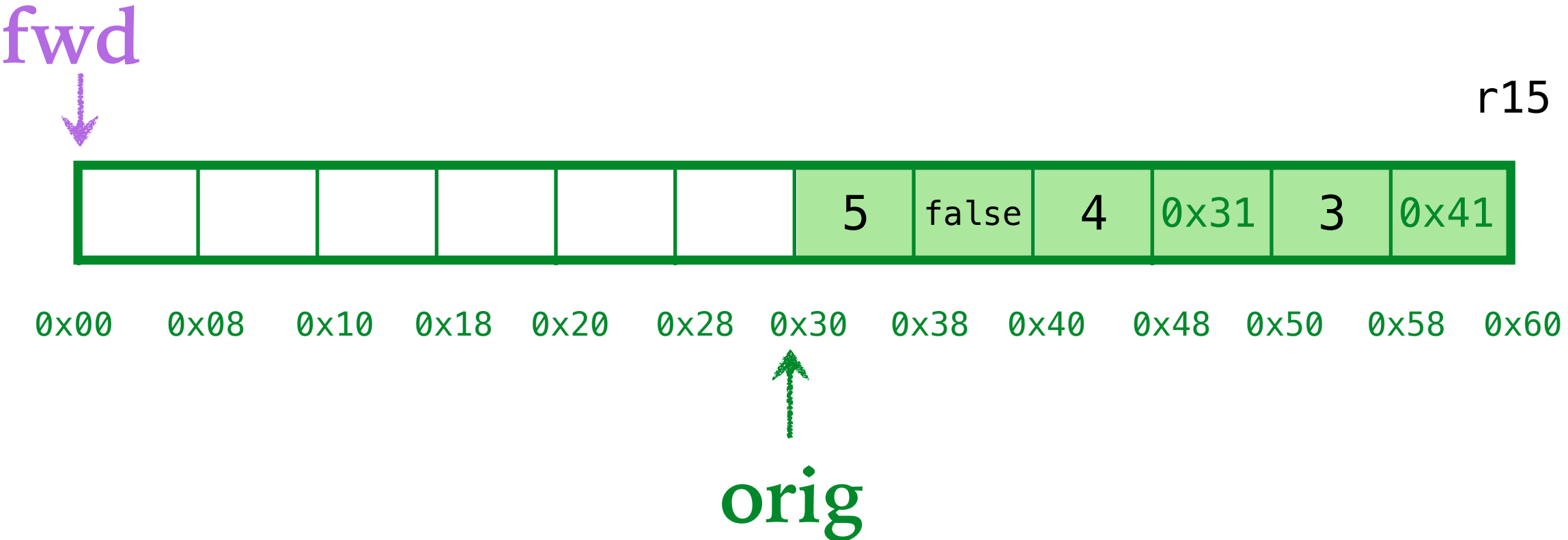
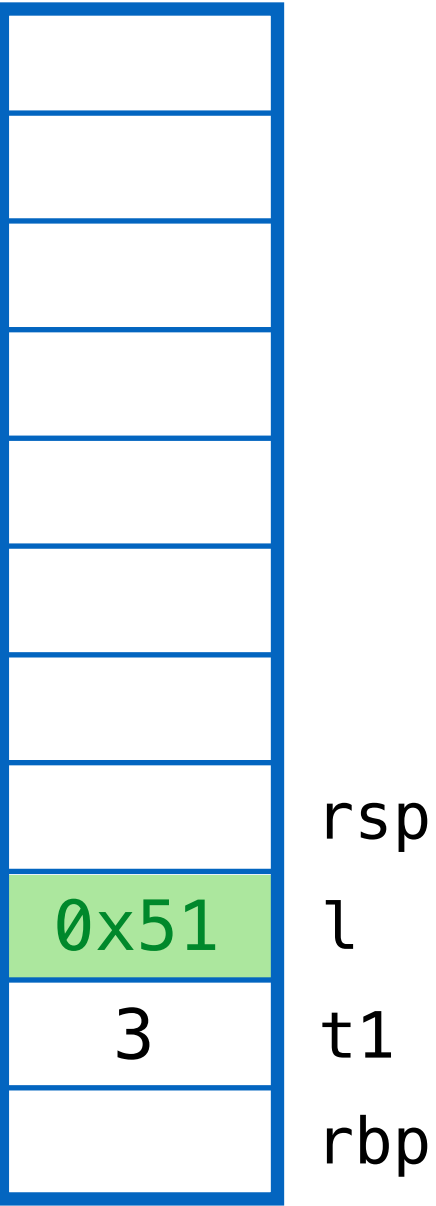
let t1 =
  let l1 = range(0, 3)
  in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)
```



2. Compute **FORWARD** addrs

ex4: recursive data

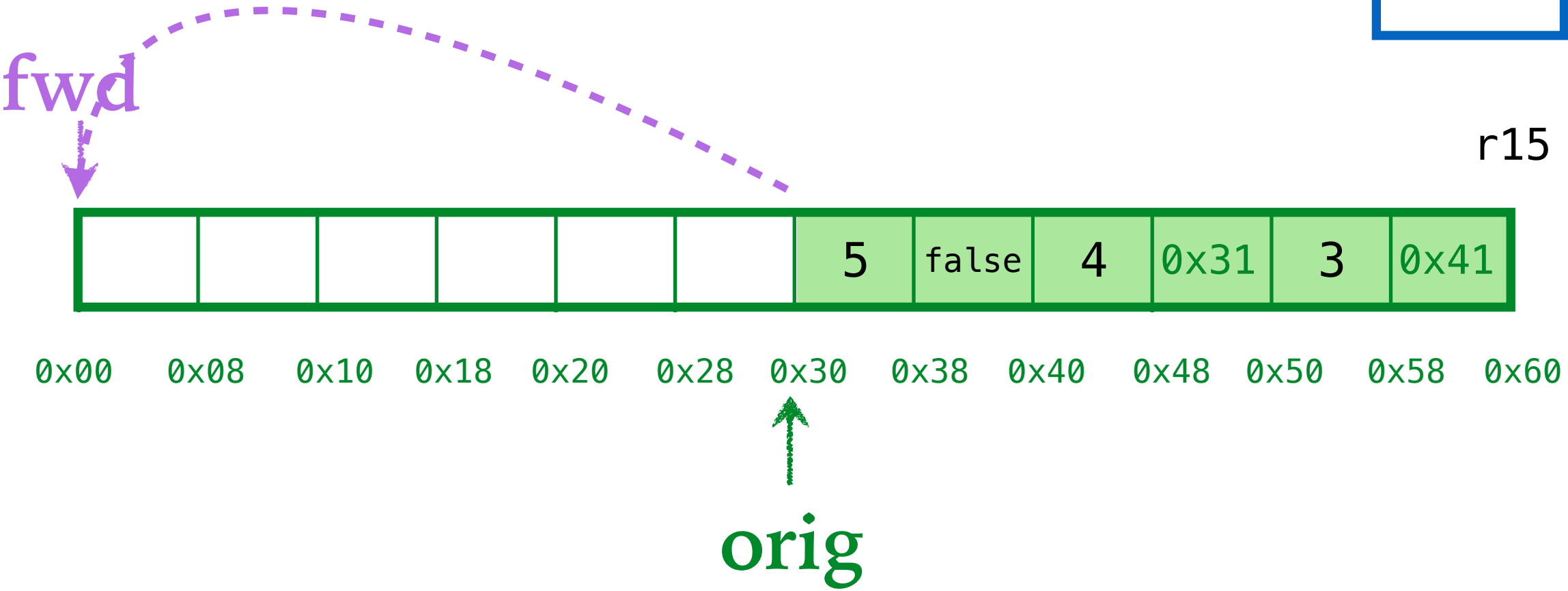
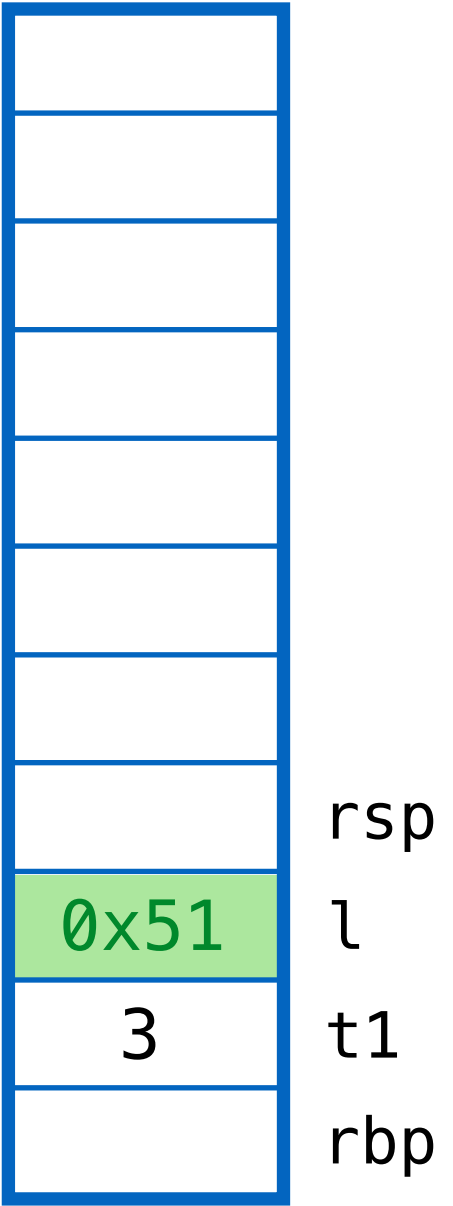
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



2. Compute FORWARD addrs

ex4: recursive data

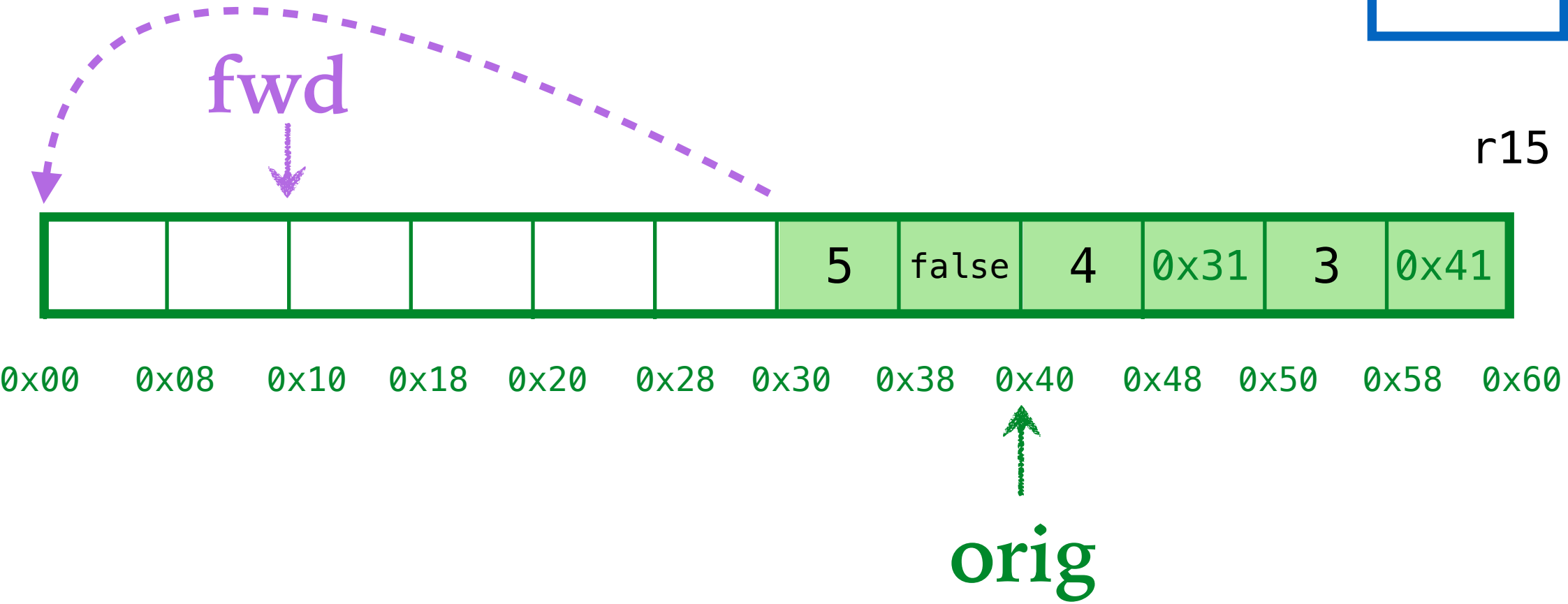
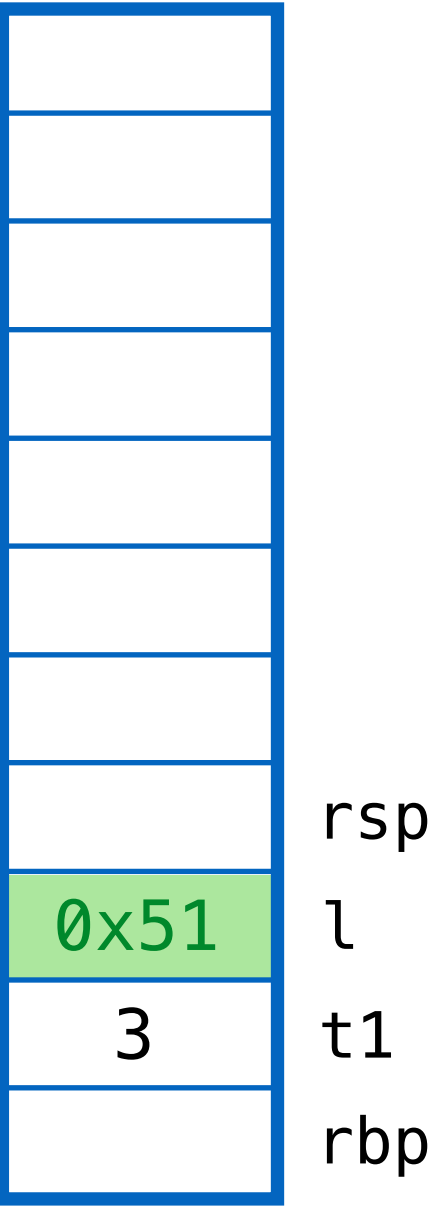
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



2. Compute FORWARD addrs

ex4: recursive data

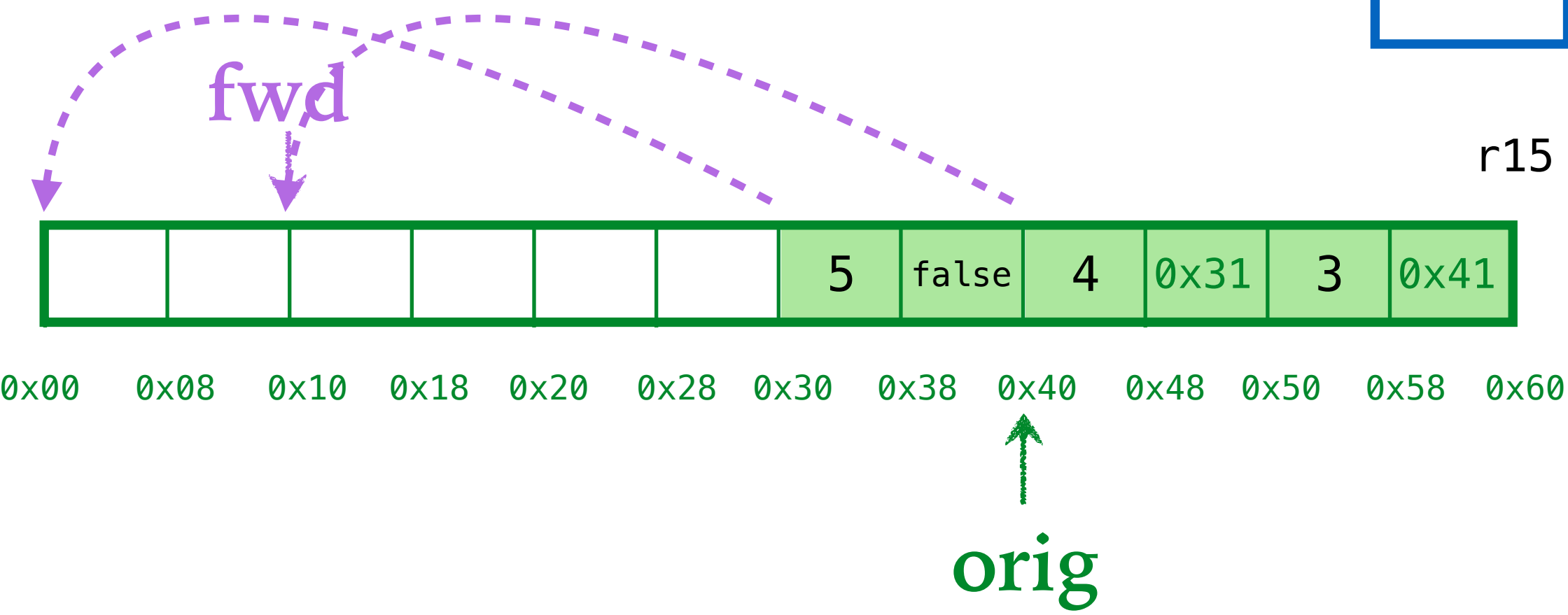
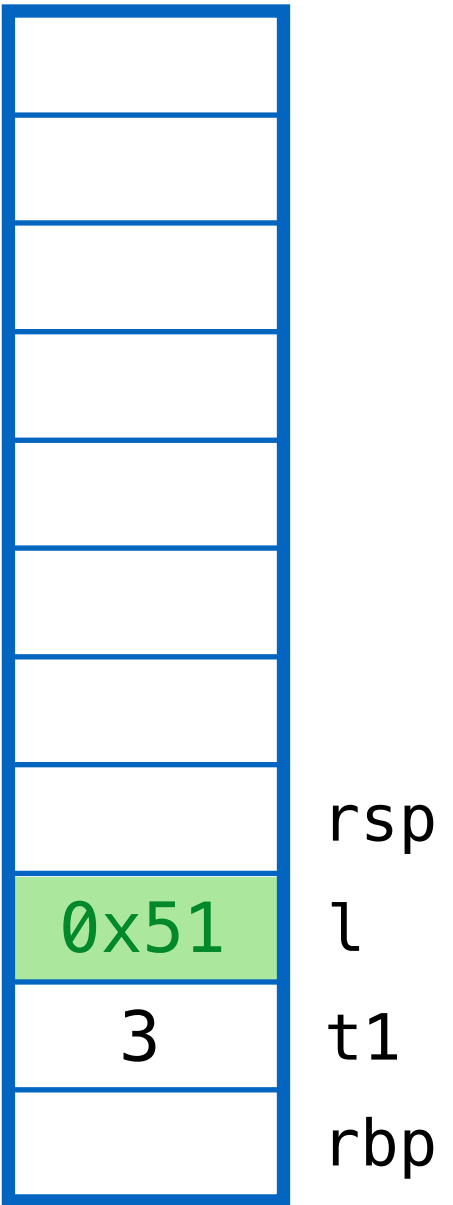
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



2. Compute FORWARD addrs

ex4: recursive data

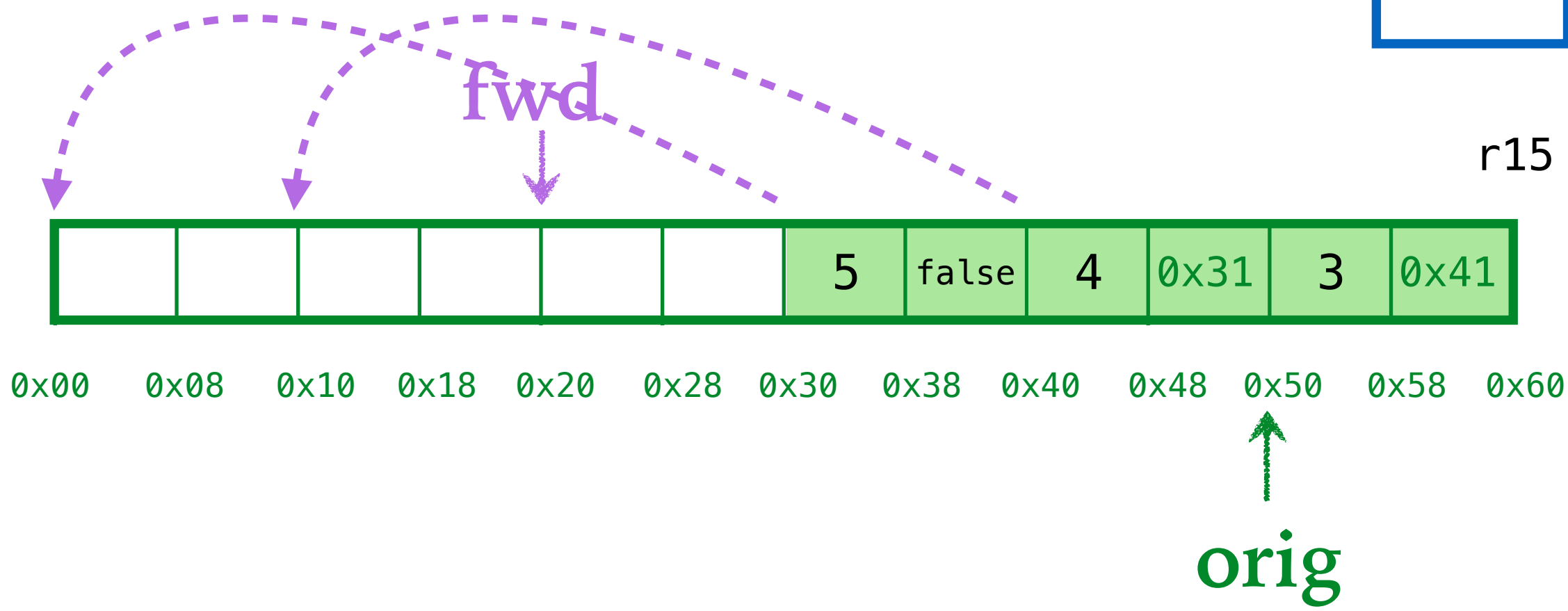
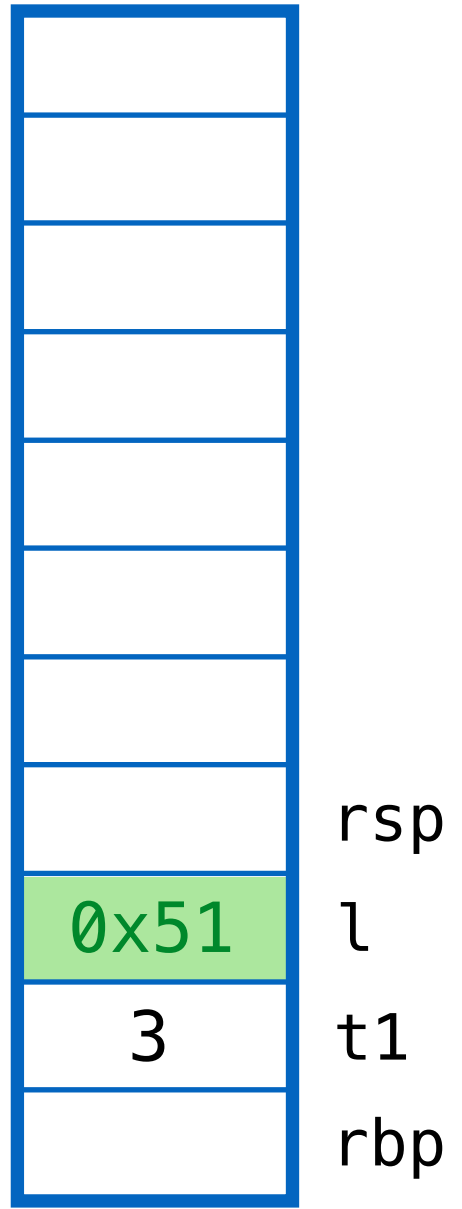
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



2. Compute FORWARD addrs

ex4: recursive data

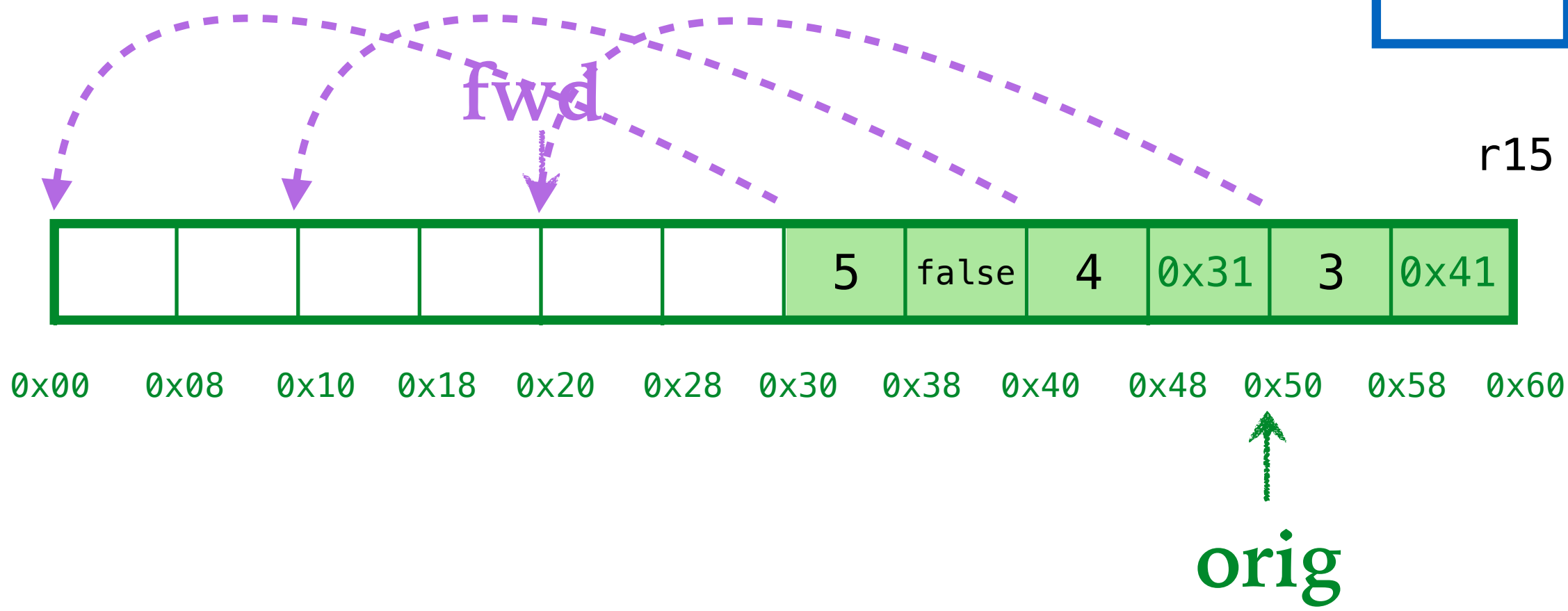
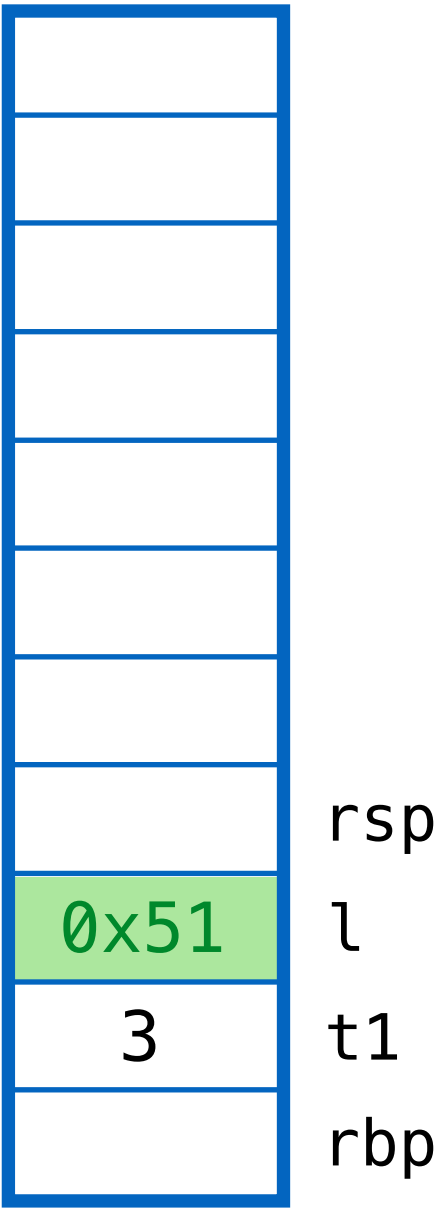
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



2. Compute FORWARD addrs

ex4: recursive data

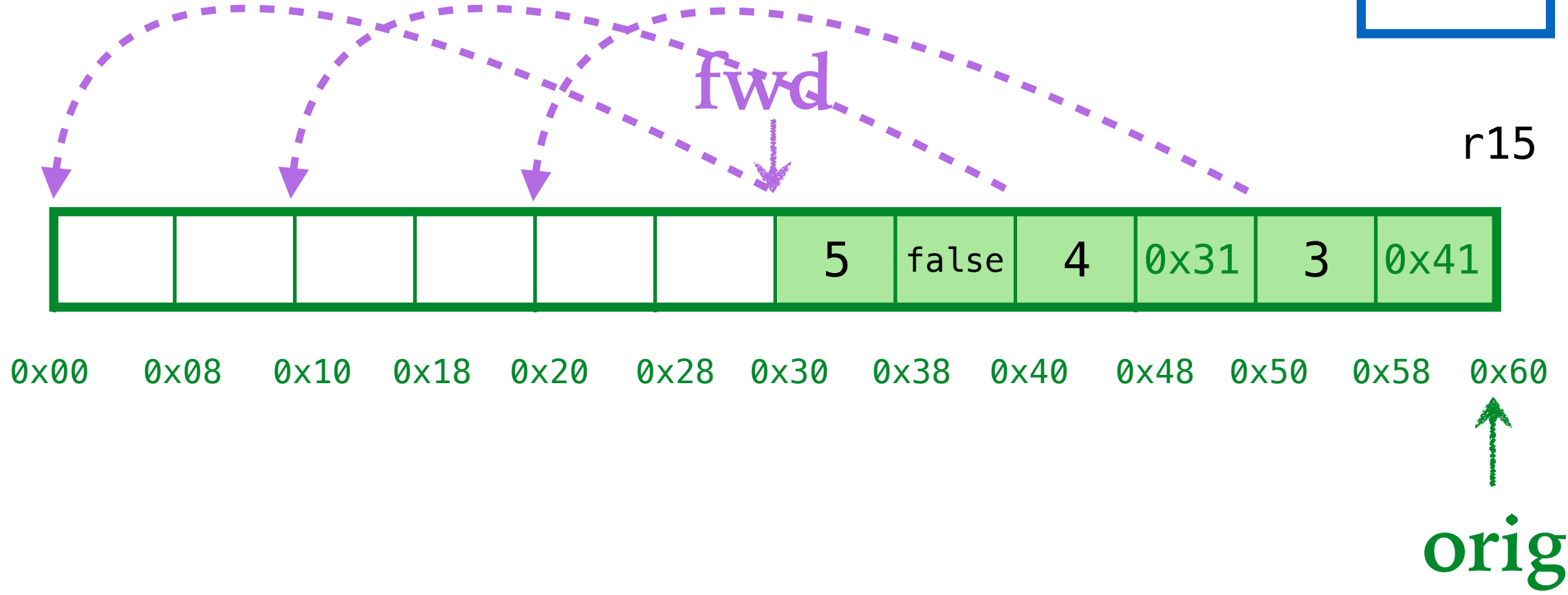
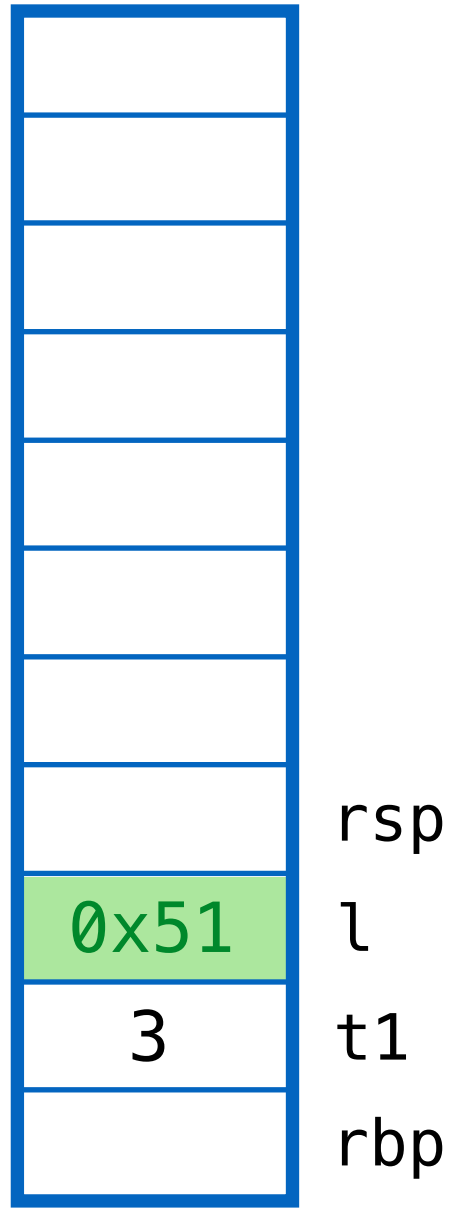
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



2. Compute FORWARD addrs

ex4: recursive data

```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



2. Compute FORWARD addrs

ex4: recursive data

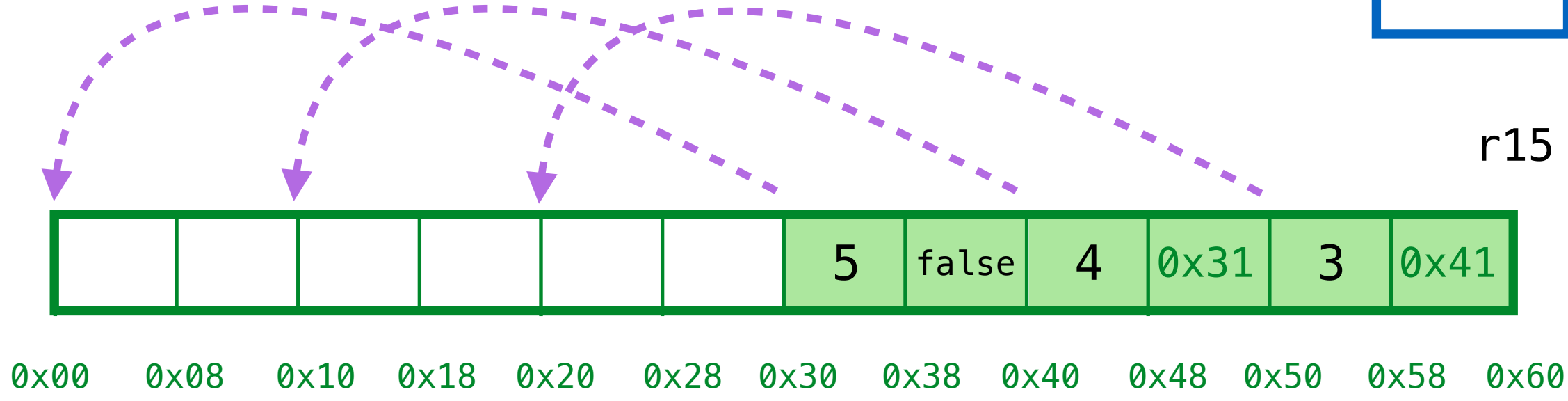
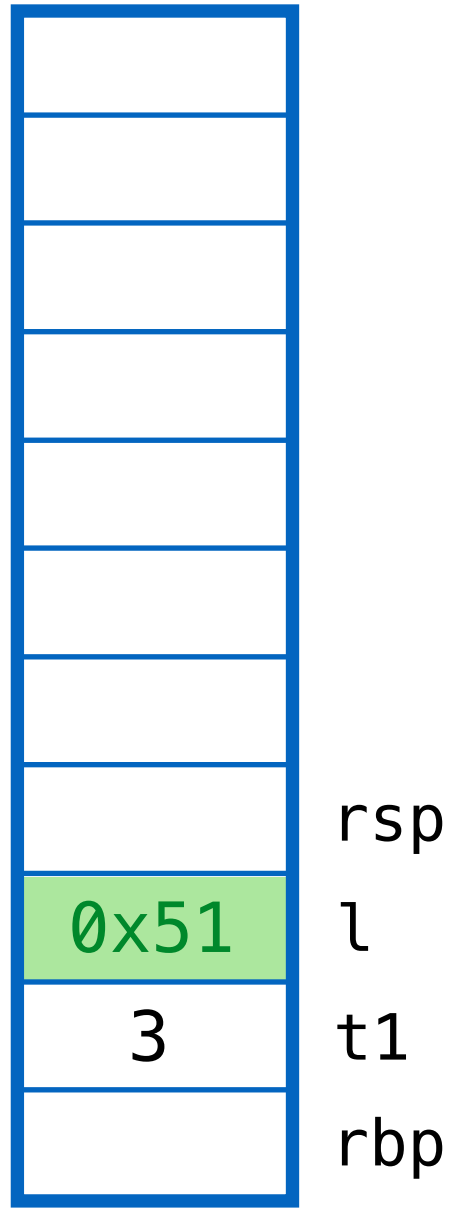
```

def range(i, j):
    if (j <= i): false else: (i,range(i+1, j))

def sum(l):
    if l == false: 0 else: l[0] + sum(l[1])

let t1 =
    let l1 = range(0, 3)
    in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)

```

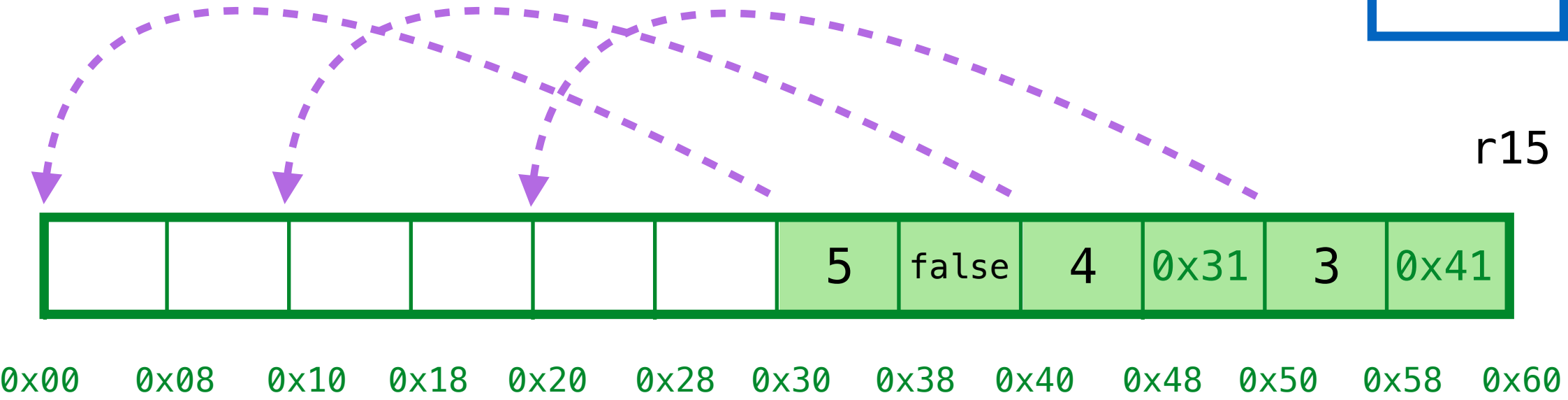
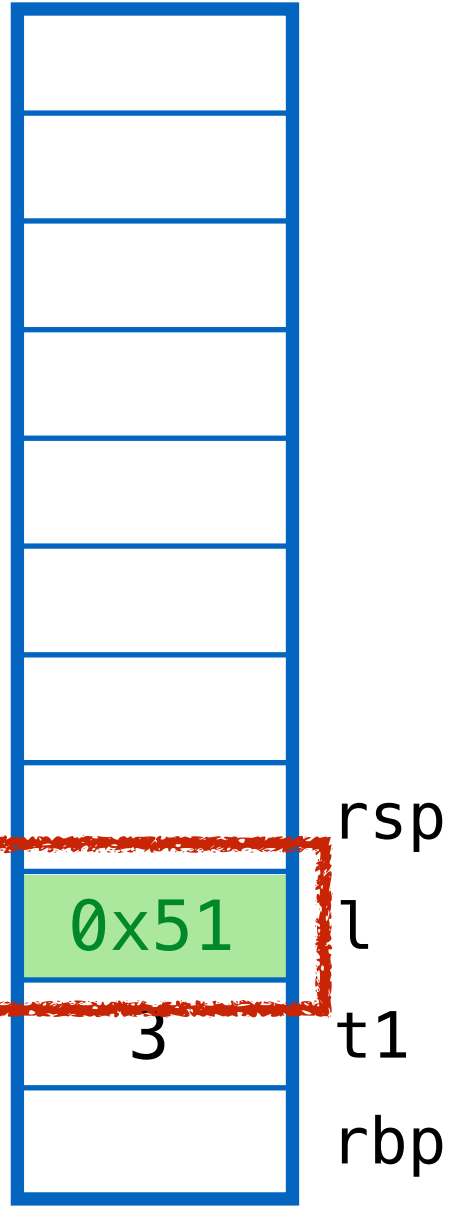


2. Compute FORWARD addrs

Where should we store the forward addrs?

ex4: recursive data

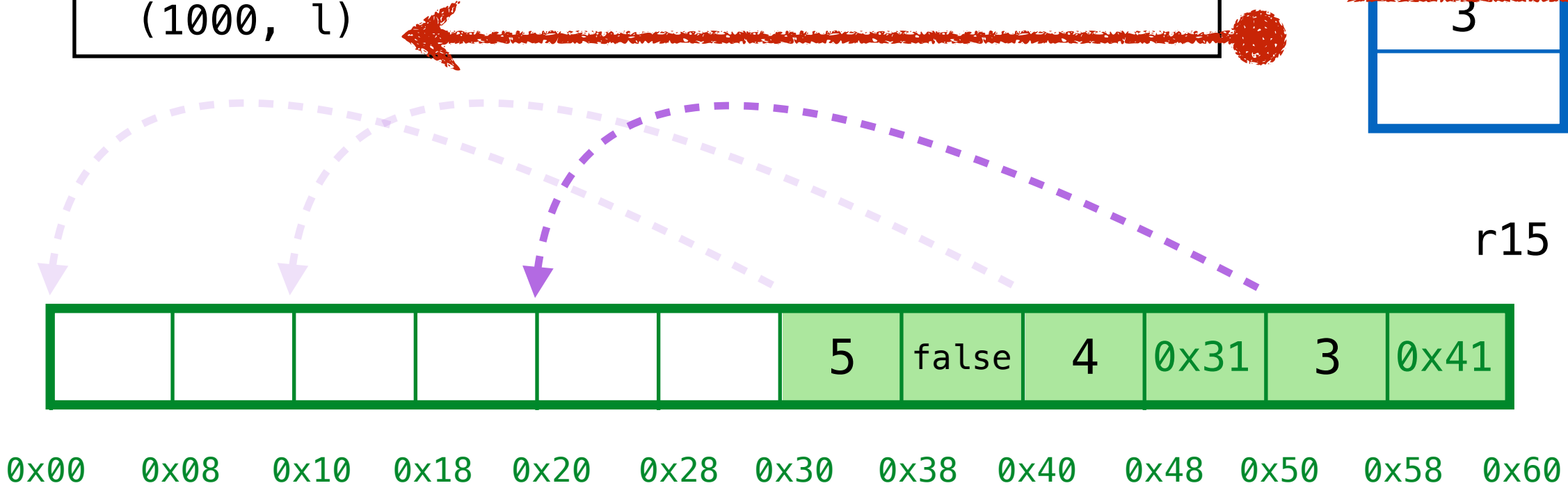
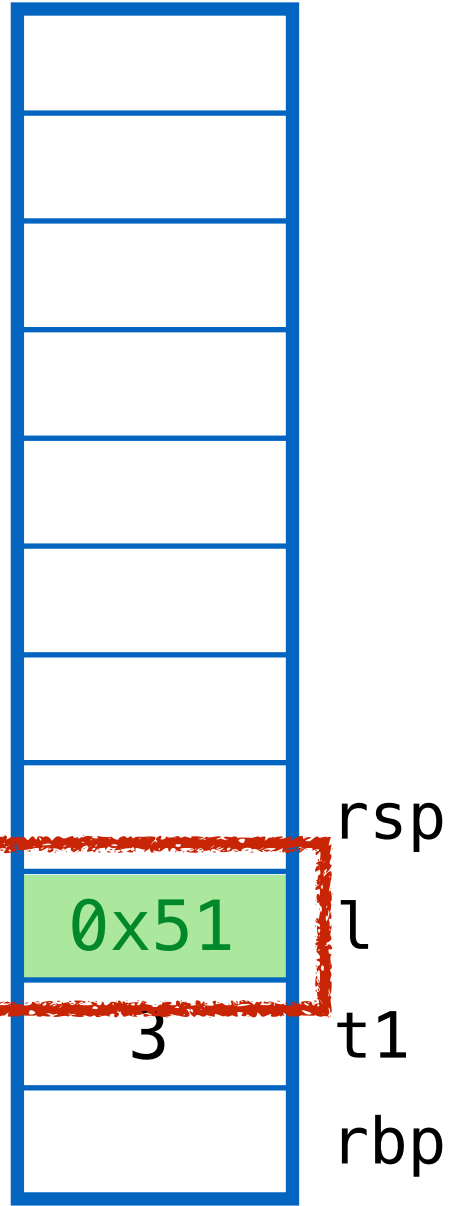
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



3. REDIRECT addr on stack

ex4: recursive data

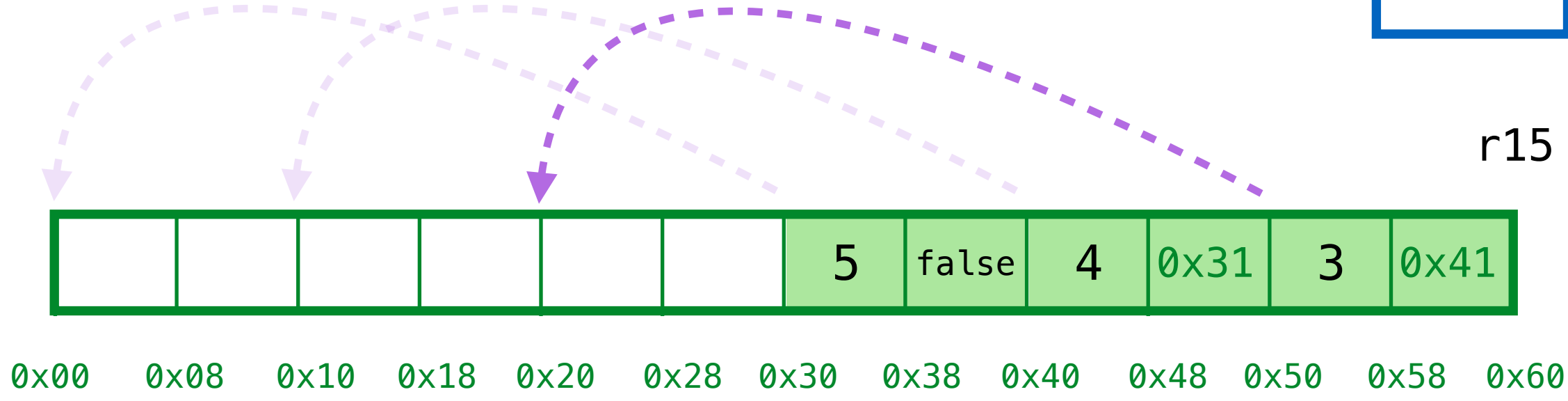
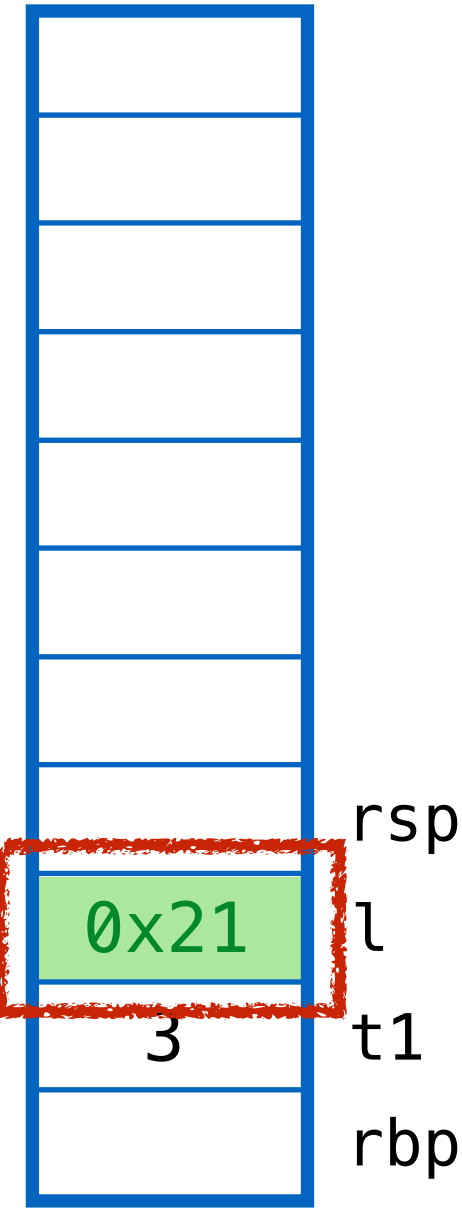
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



3. REDIRECT addr on stack

ex4: recursive data

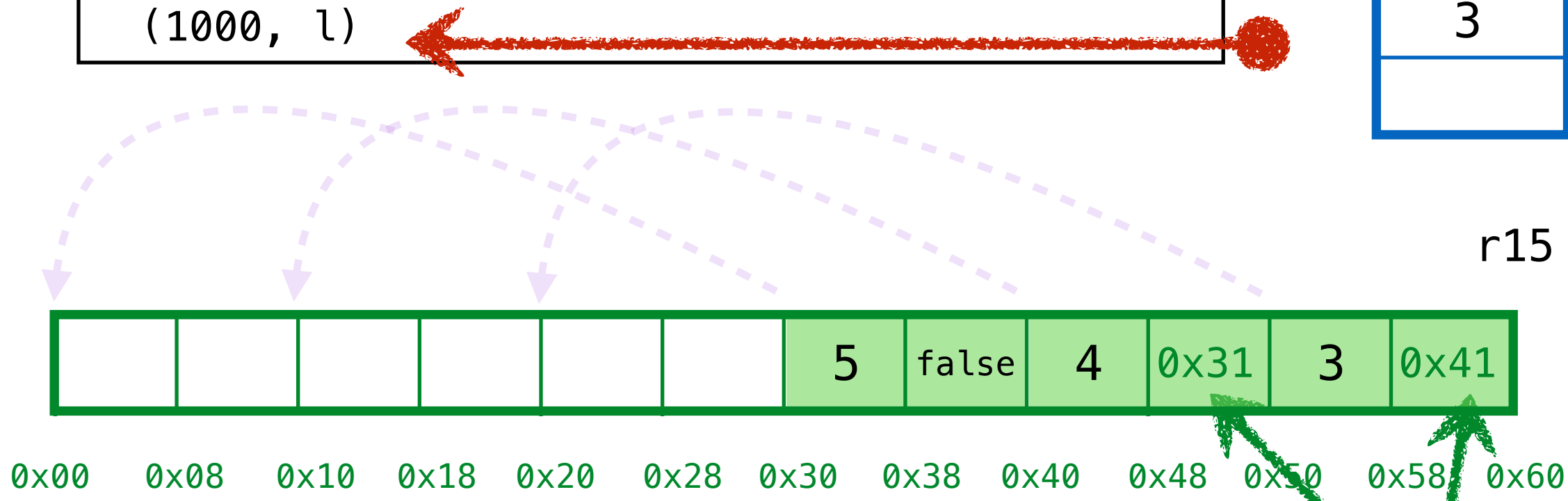
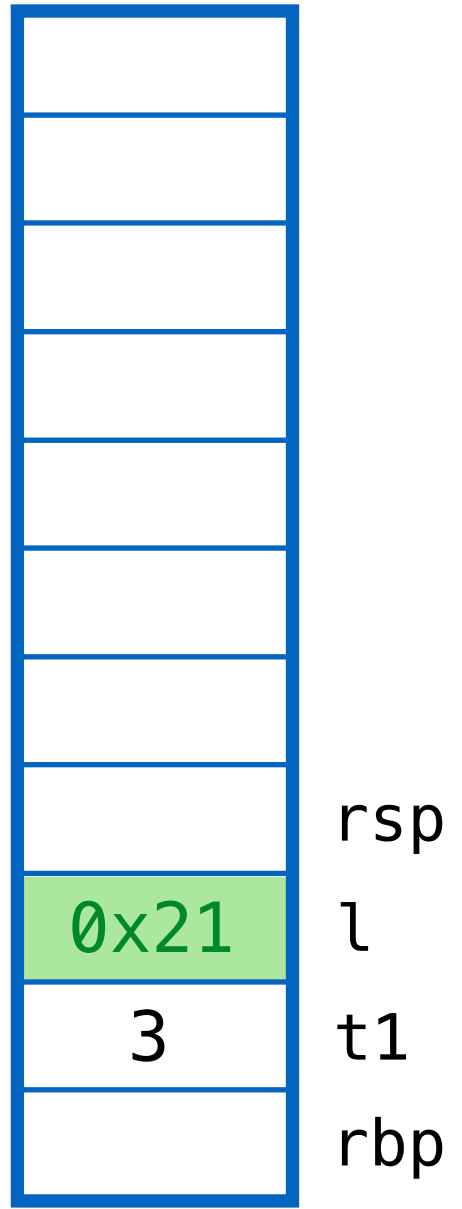
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
    , l = range(t1, t1 + 3)  
in  
(1000, l)
```



3. REDIRECT addr on stack

ex4: recursive data

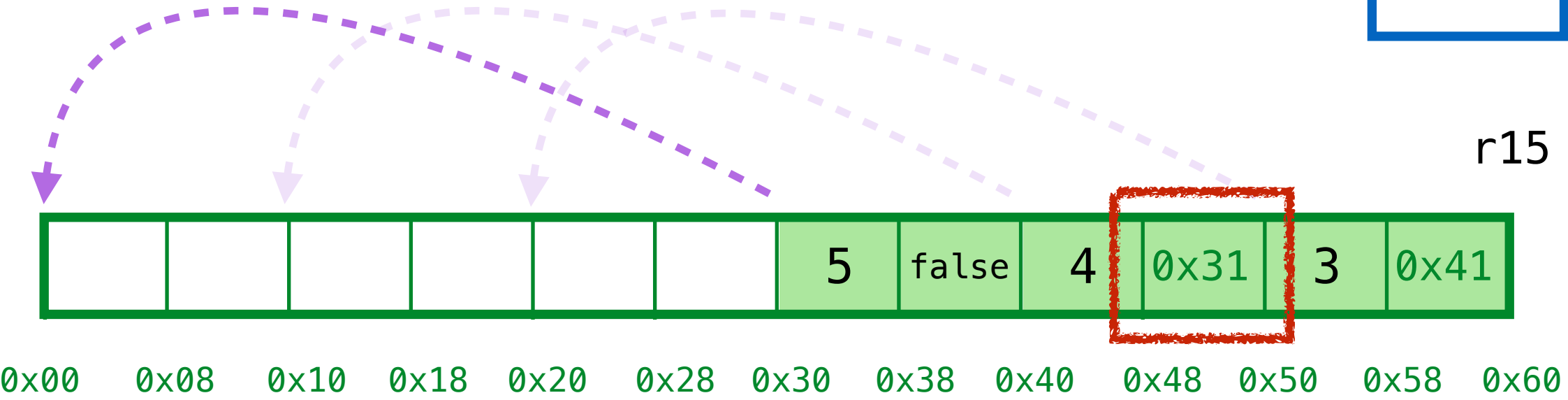
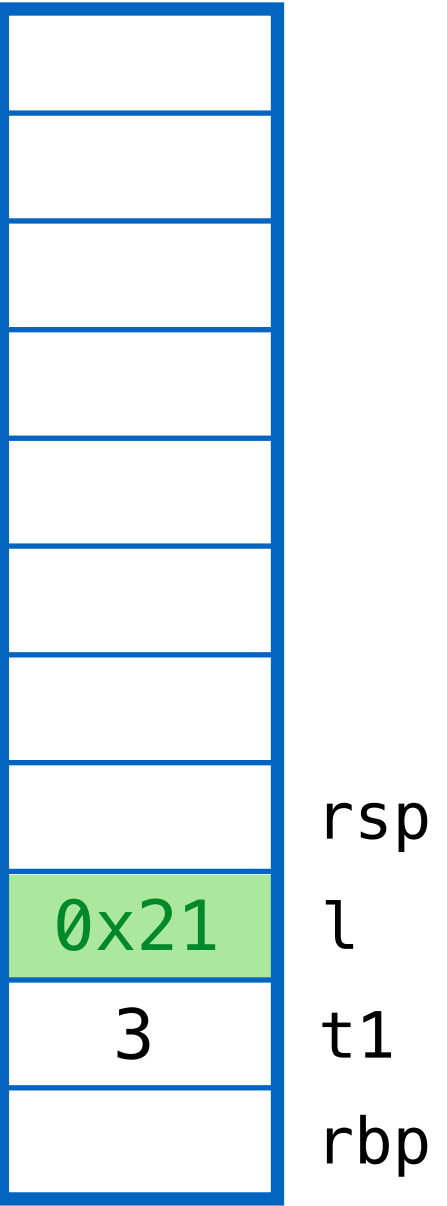
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



3. REDIRECT addr on stack and heap!

ex4: recursive data

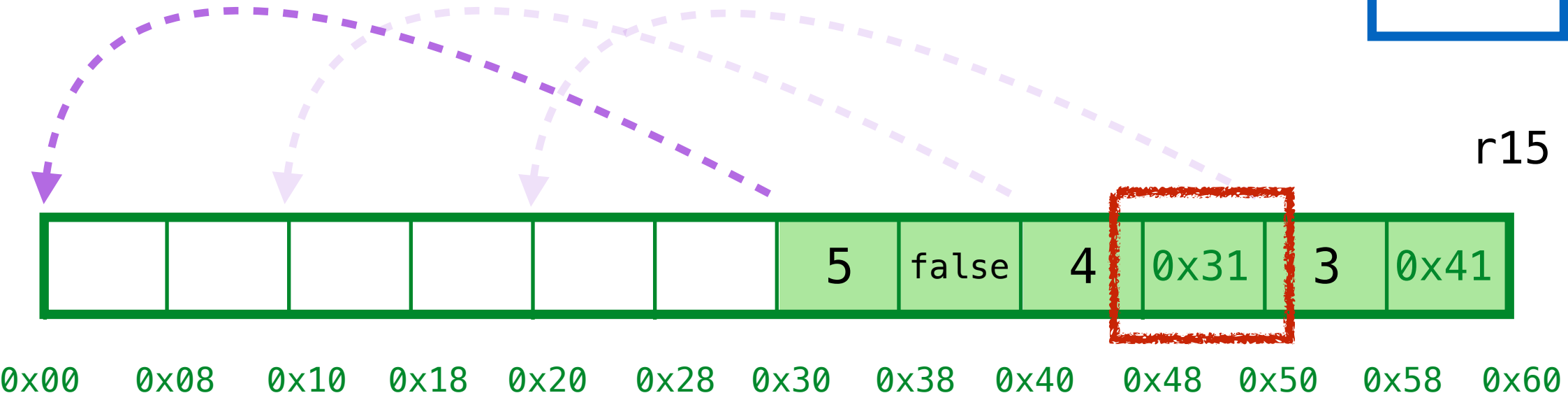
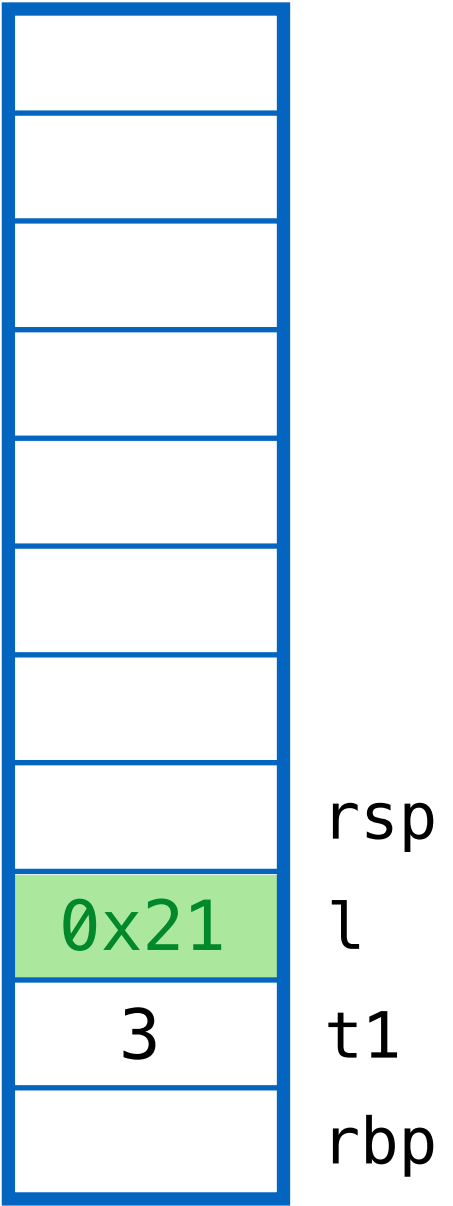
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



3. REDIRECT addrs on stack and heap!

ex4: recursive data

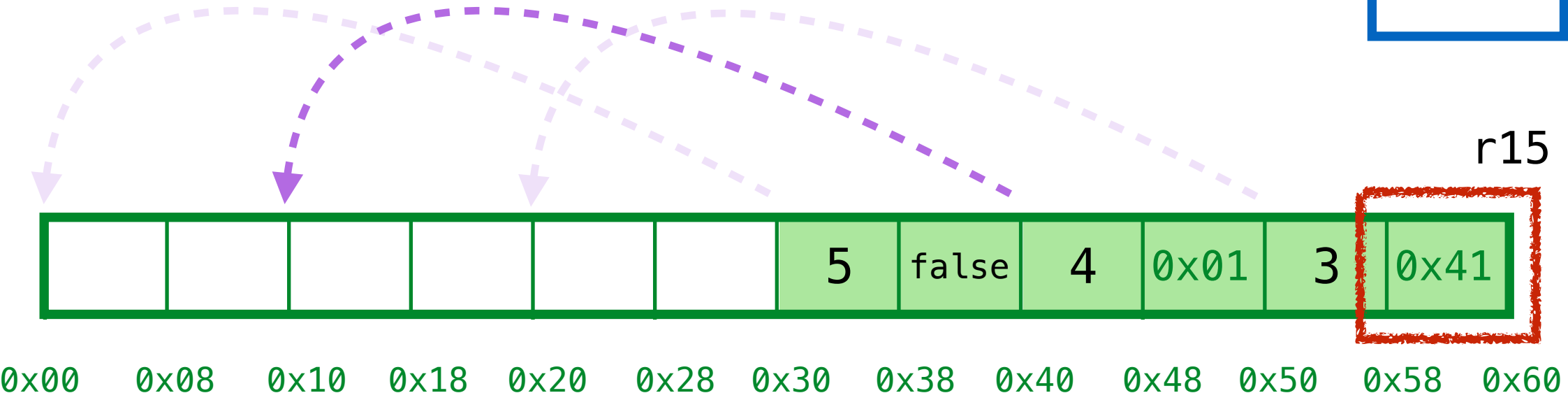
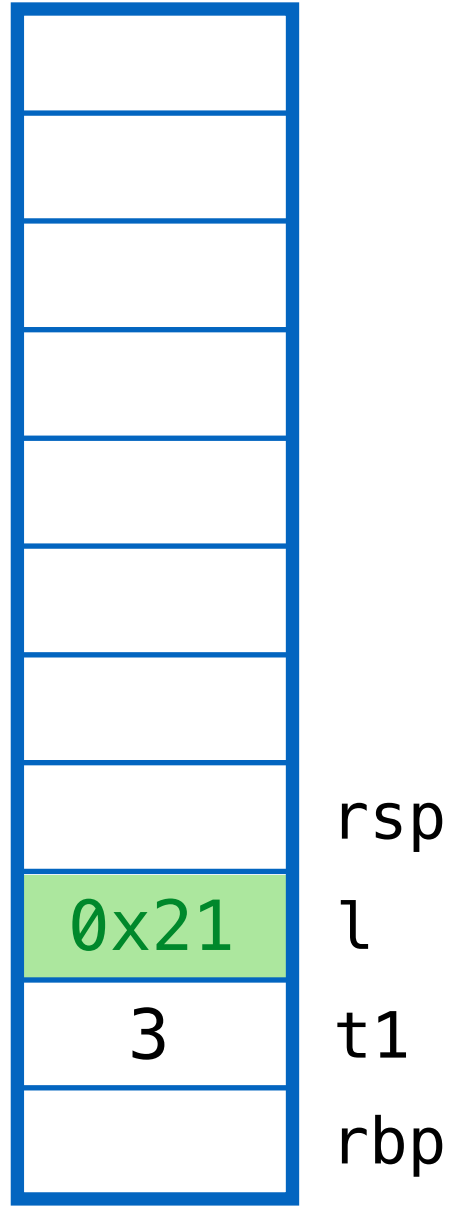
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
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in  
(1000, l)
```



3. REDIRECT addr on stack and heap!

ex4: recursive data

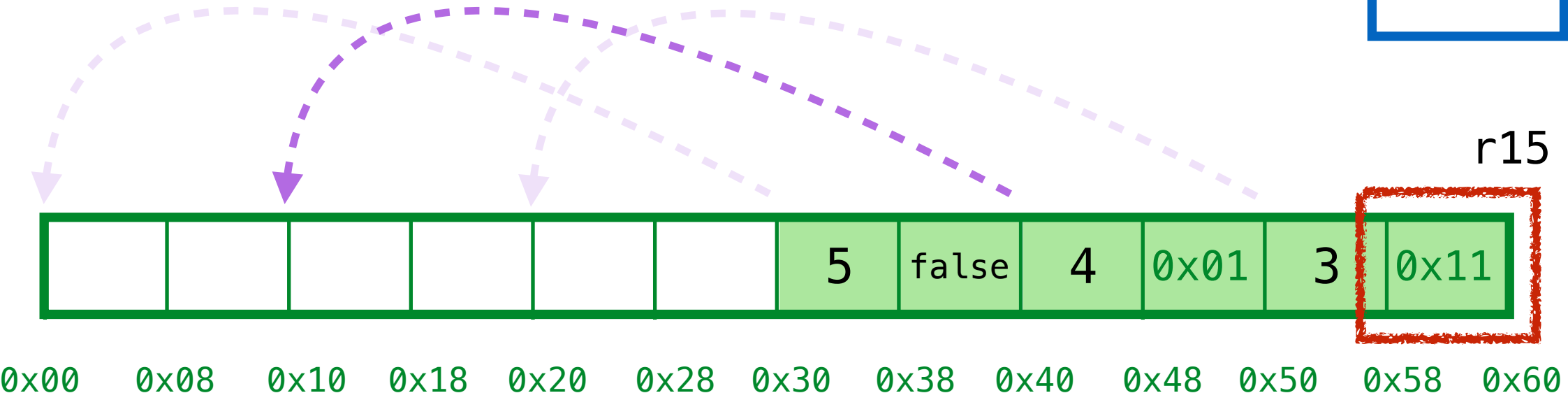
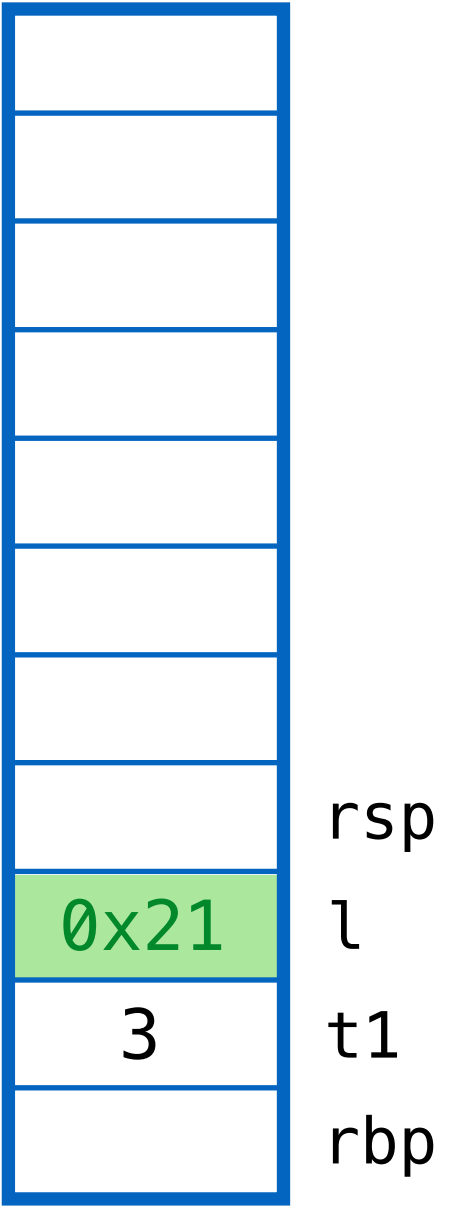
```
def range(i, j):  
    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
    if l == false: 0 else: l[0] + sum(l[1])  
  
let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



3. REDIRECT addr on stack and heap!

ex4: recursive data

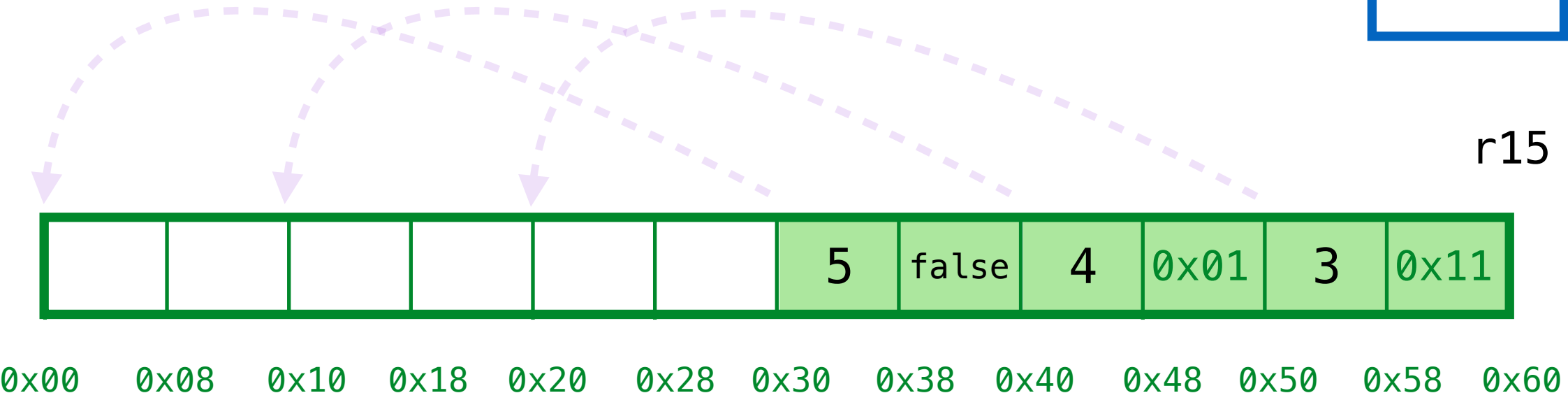
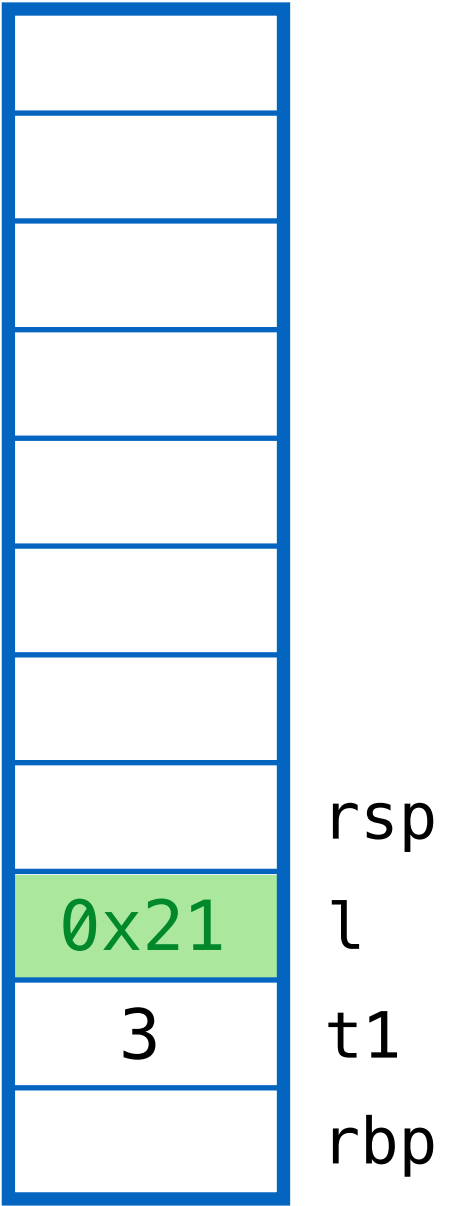
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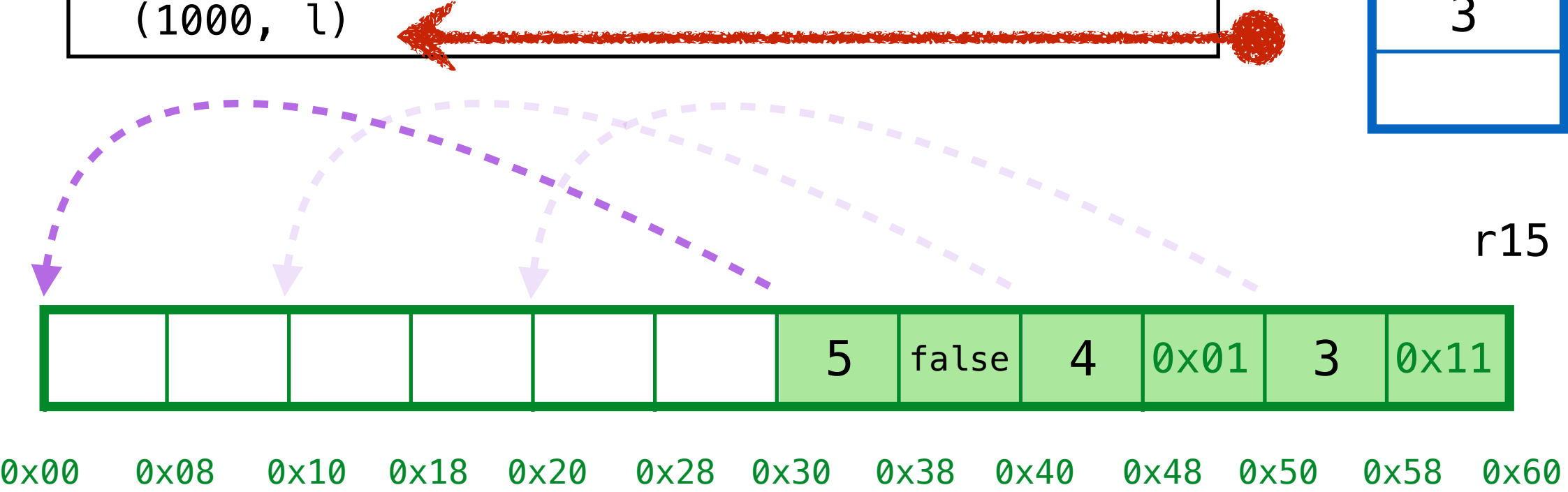
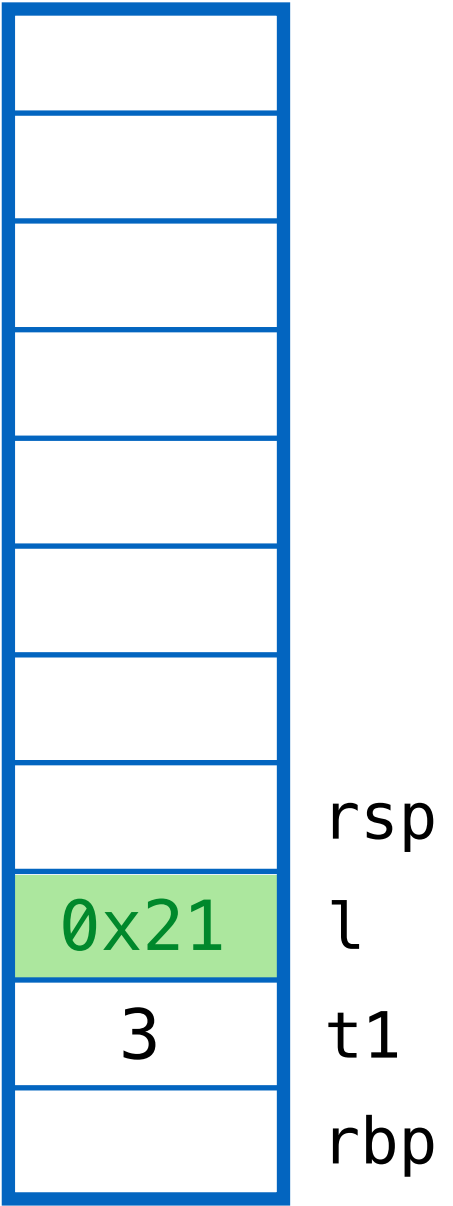


4. COMPACT cells on heap

Copy cell to forward addr!

ex4: recursive data

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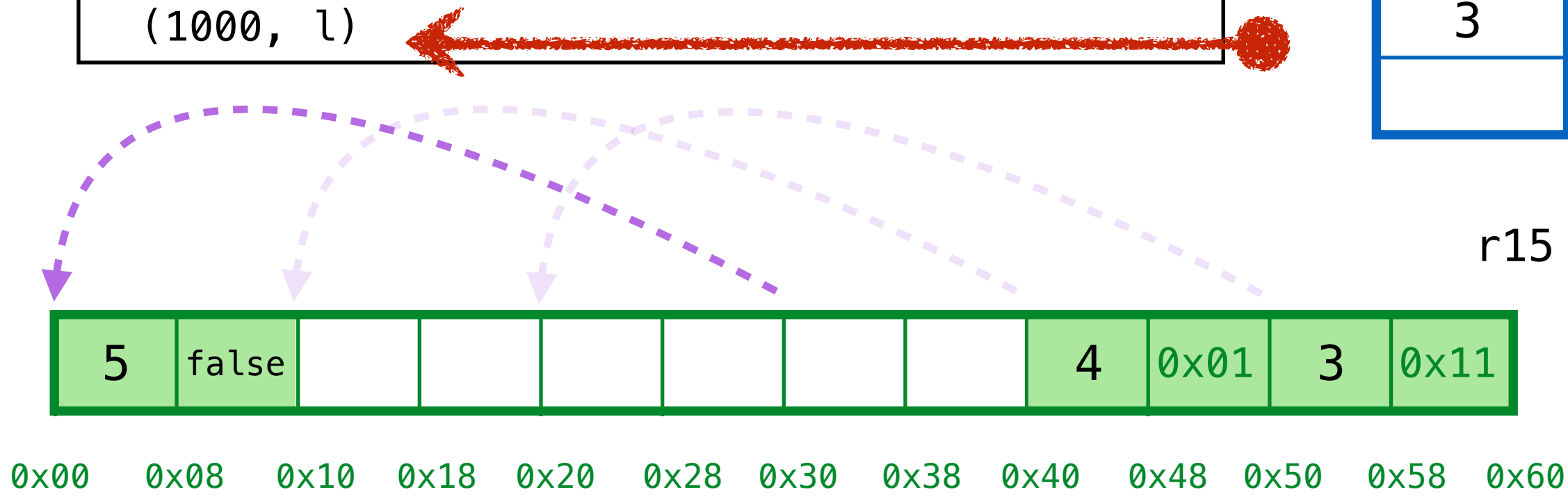
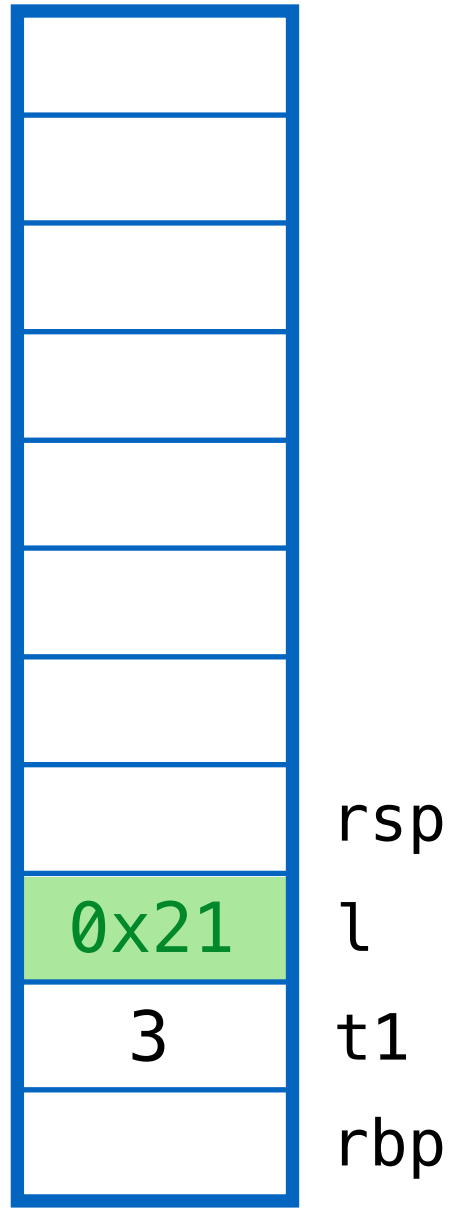


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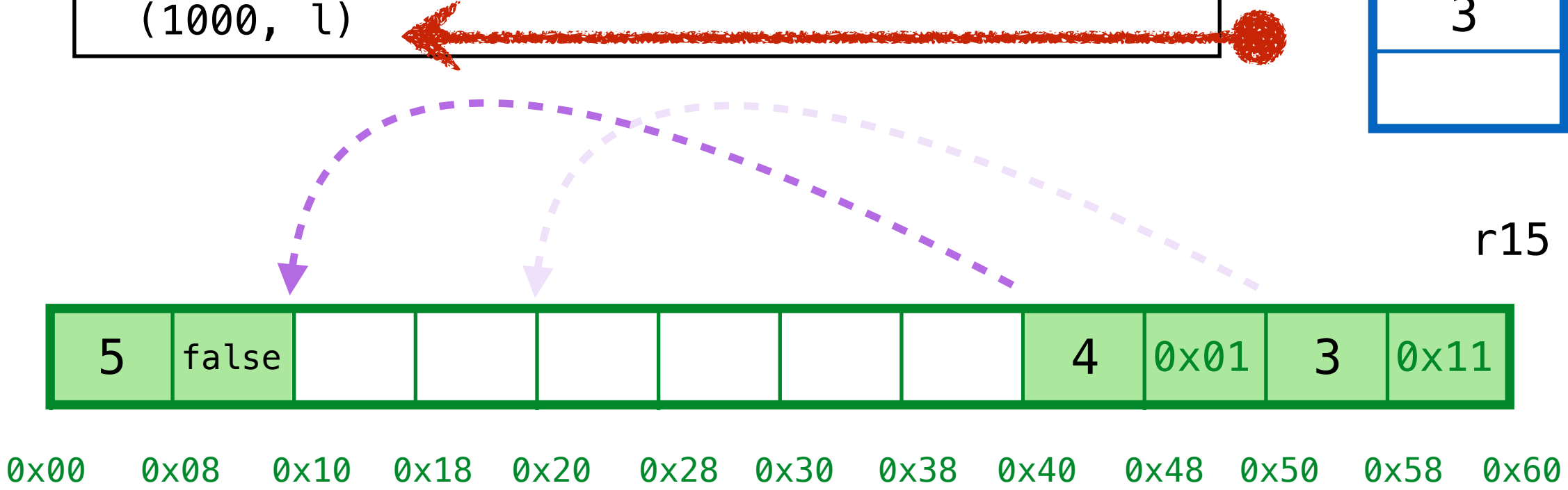
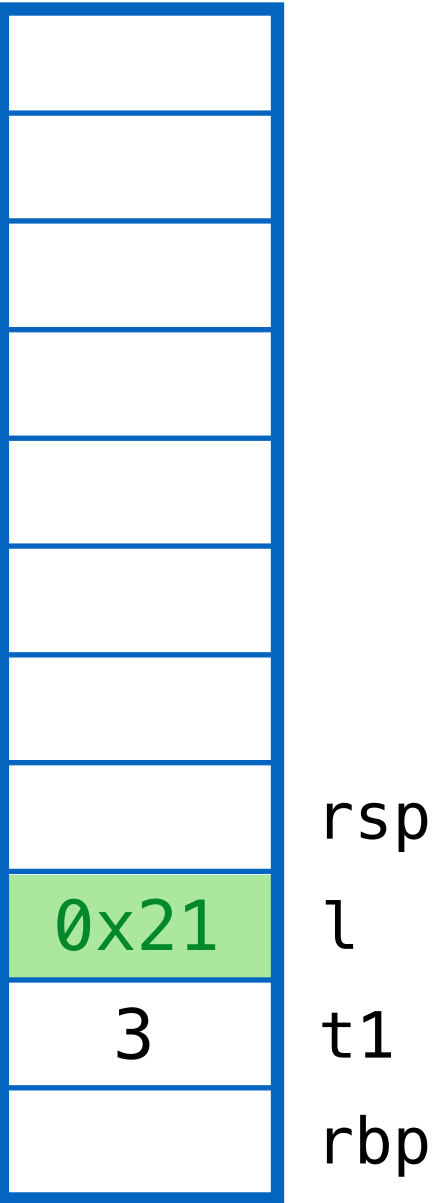
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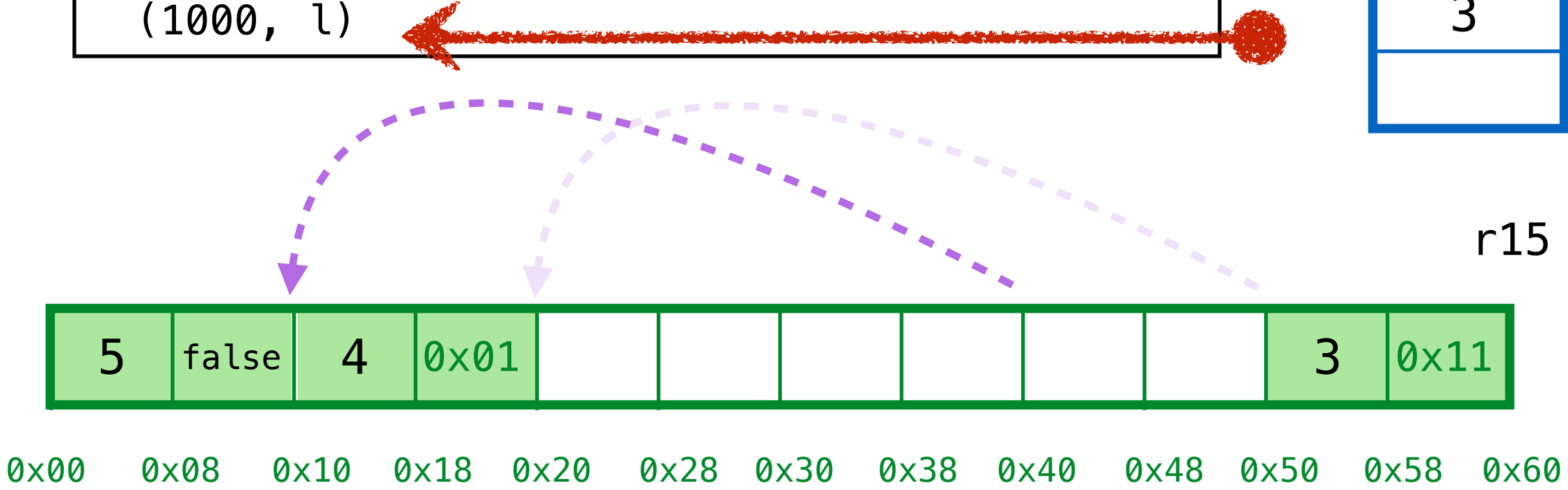
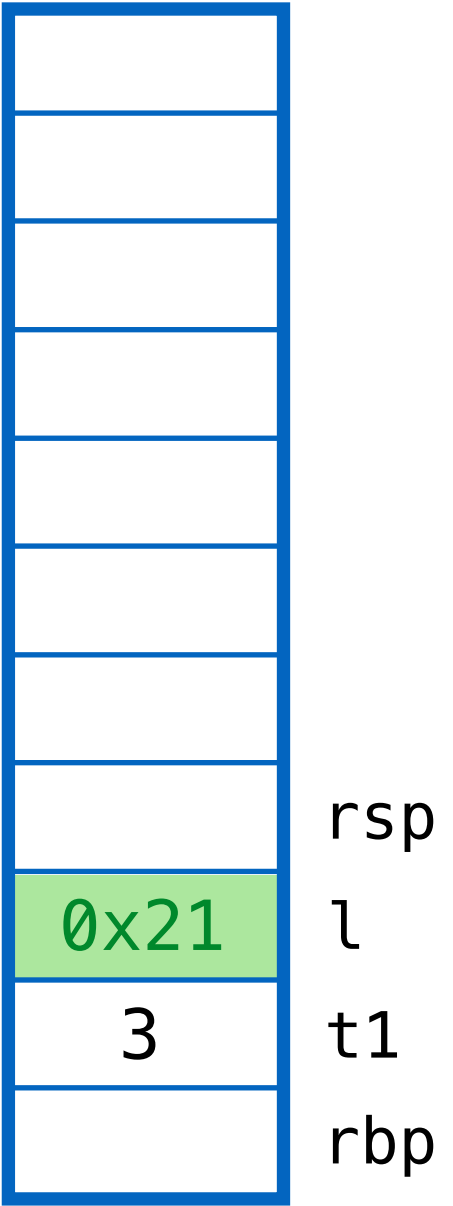
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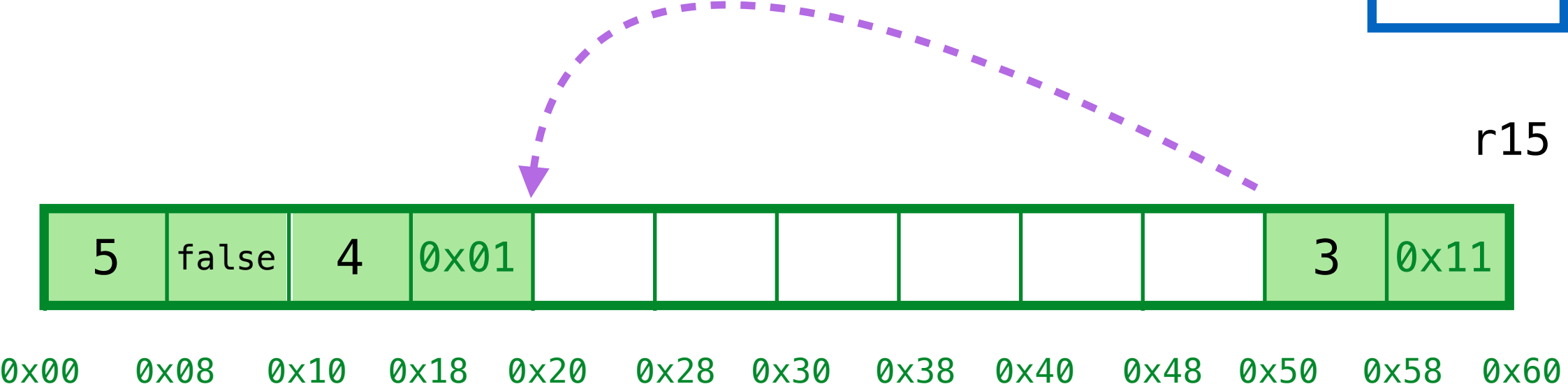
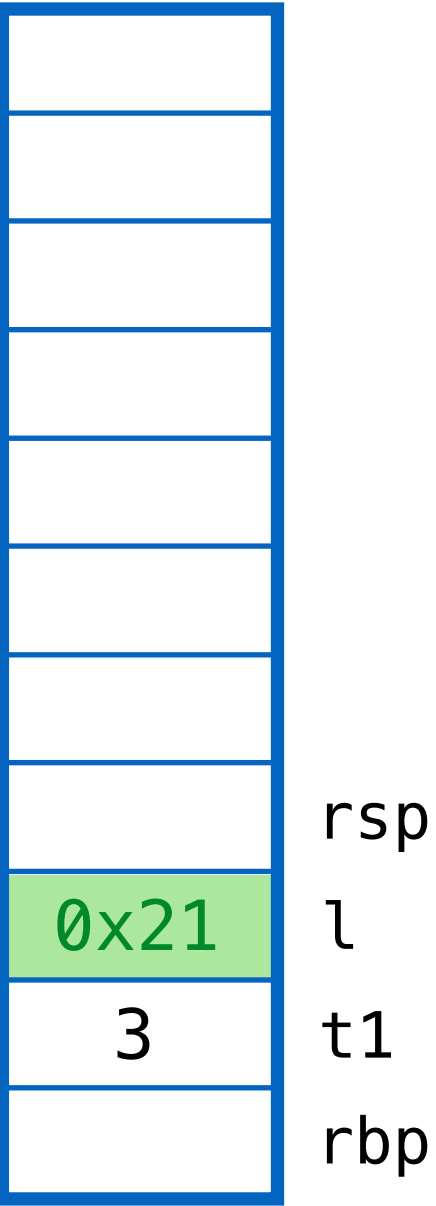
```

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def sum(l):
  if l == false: 0 else: l[0] + sum(l[1])

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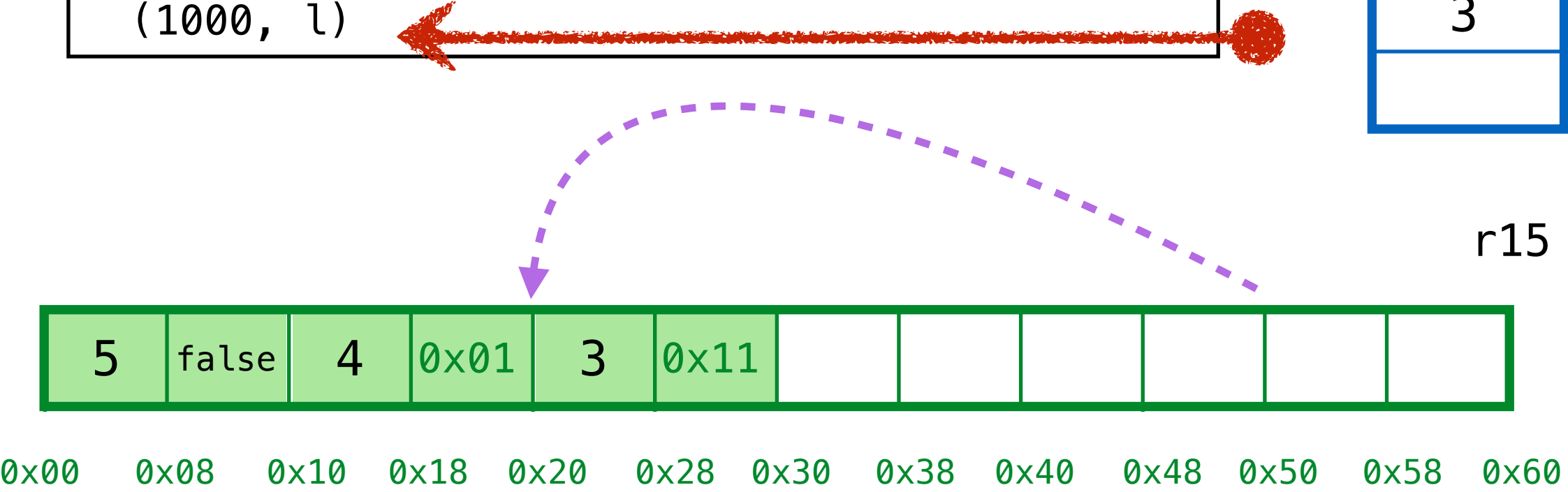
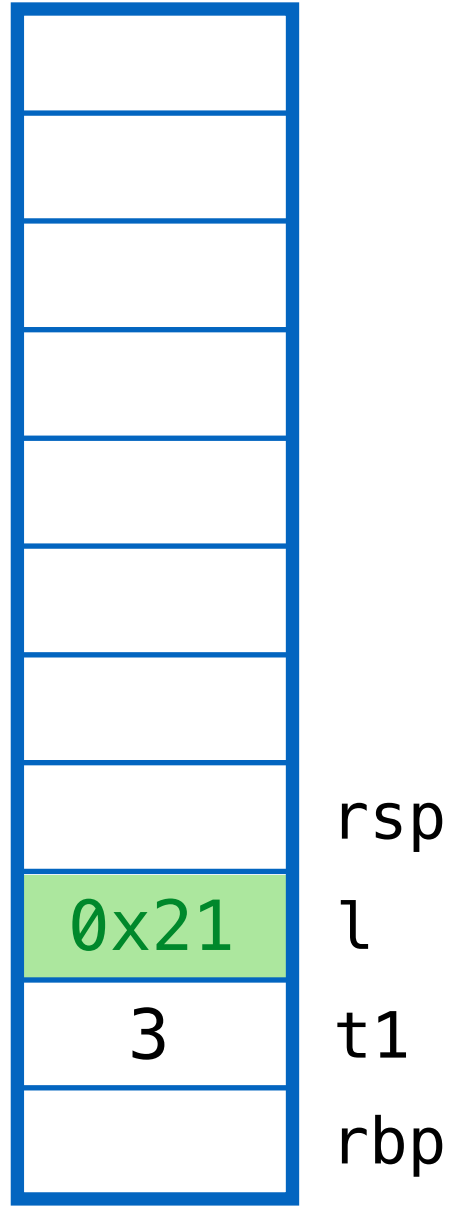
```



4. COMPACT cells on heap
 Copy cell to forward addr!

ex4: recursive data

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    if (j <= i): false else: (i,range(i+1, j))  
  
def sum(l):  
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let t1 =  
    let l1 = range(0, 3)  
    in sum(l1)  
, l = range(t1, t1 + 3)  
in  
(1000, l)
```



4. COMPACT cells on heap
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ex4: recursive data

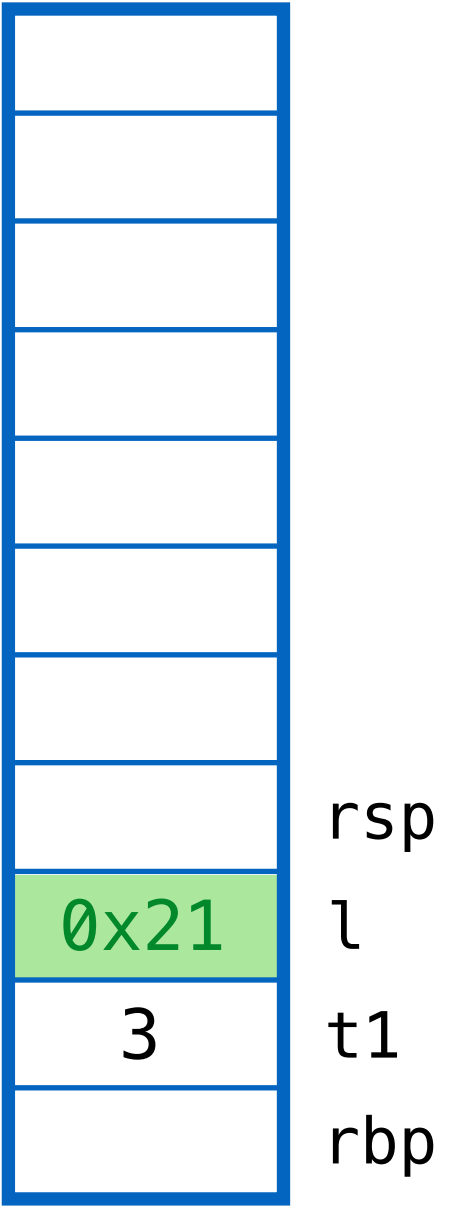
```

def range(i, j):
  if (j <= i): false else: (i,range(i+1, j))

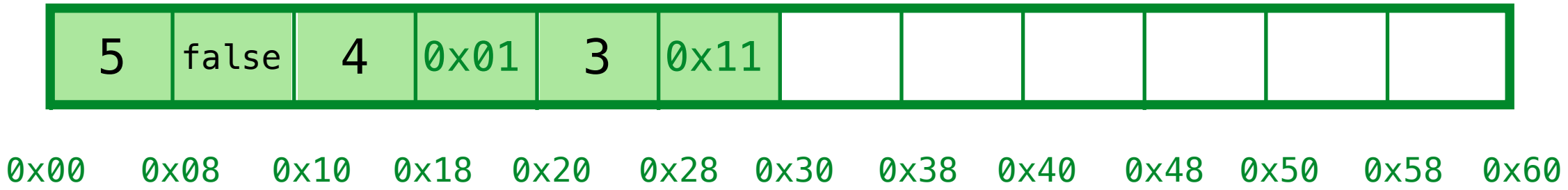
def sum(l):
  if l == false: 0 else: l[0] + sum(l[1])

let t1 =
  let l1 = range(0, 3)
  in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)

```



r15



GC Complete!
 Have space for (1000, l)

ex4: recursive data

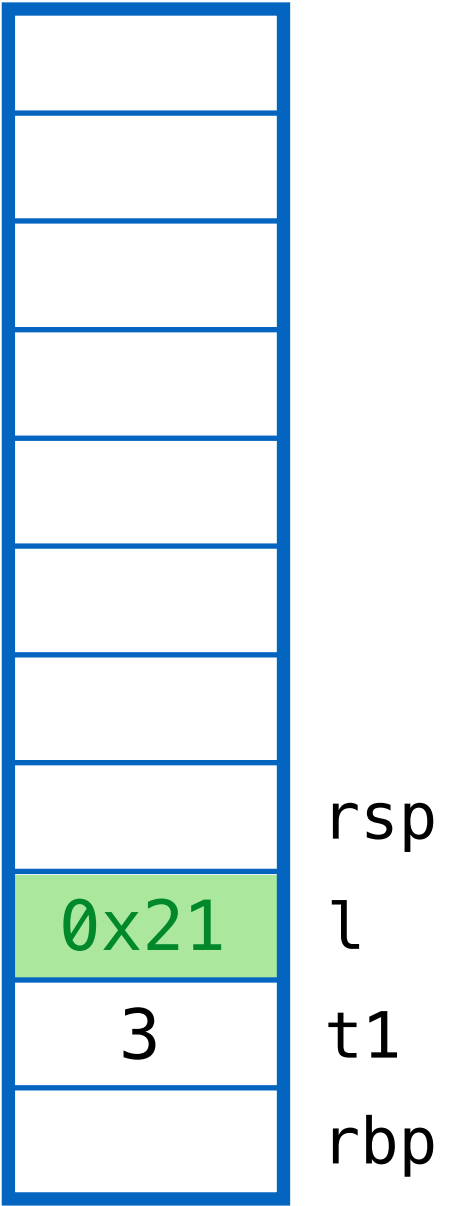
```

def range(i, j):
  if (j <= i): false else: (i, range(i+1, j))

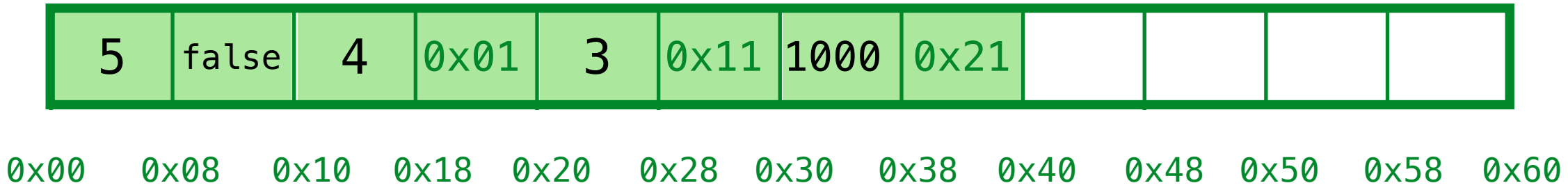
def sum(l):
  if l == false: 0 else: l[0] + sum(l[1])

let t1 =
  let l1 = range(0, 3)
  in sum(l1)
, l = range(t1, t1 + 3)
in
(1000, l)

```



r15



GC Complete!
 Have space for (1000, l)

ex4: recursive data

QUIZ: What should `print(0x21)` show?

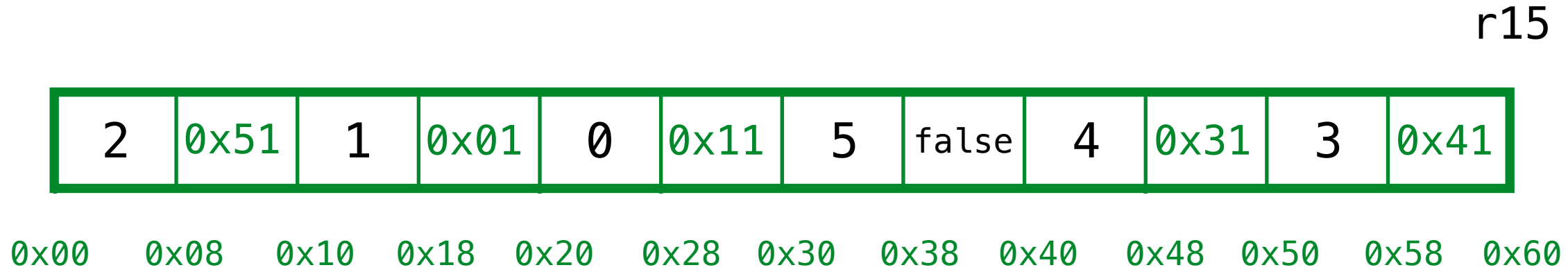
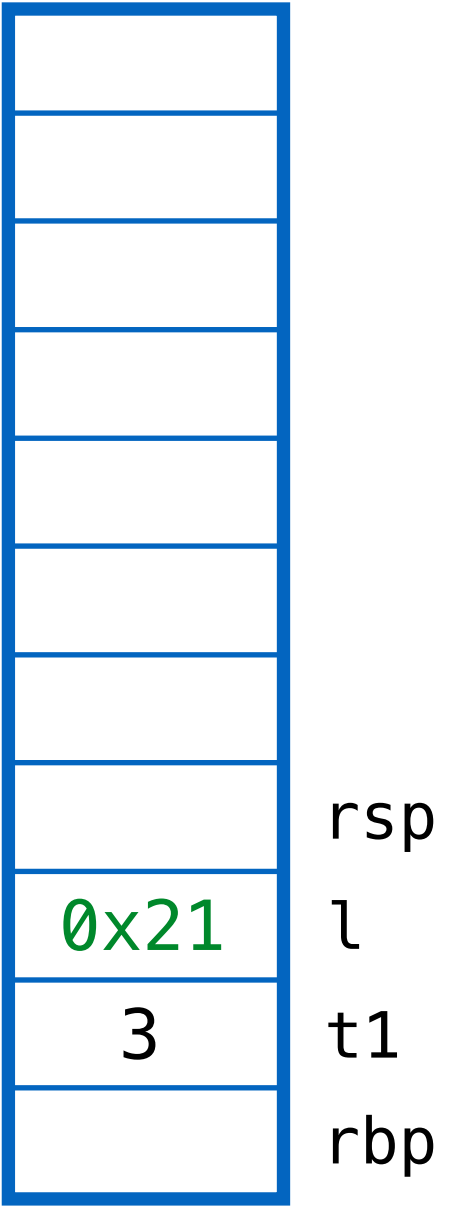
(A) (0, (1, (2, false)))

(B) (3, (4, (5, false)))

(C) (0, (1, (2, (3, (4, (5, false))))))

(D) (3, (4, (5, (0, (1, (2, false))))))

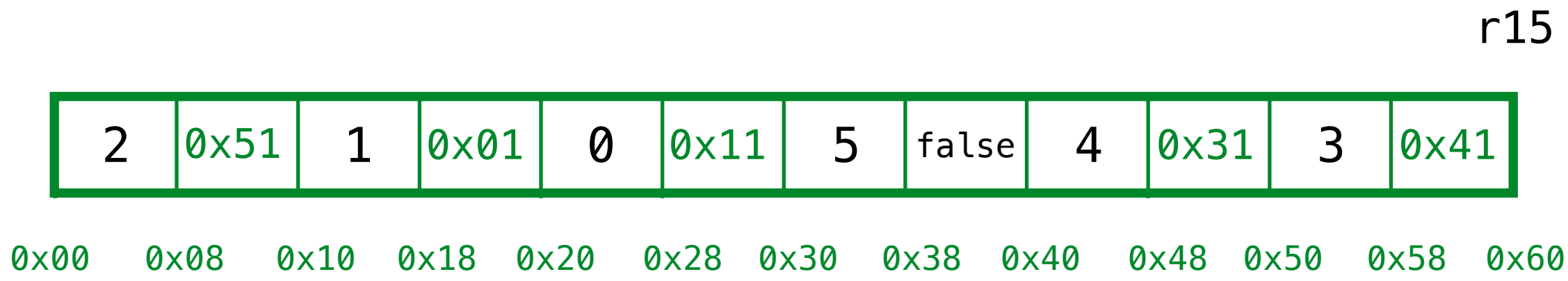
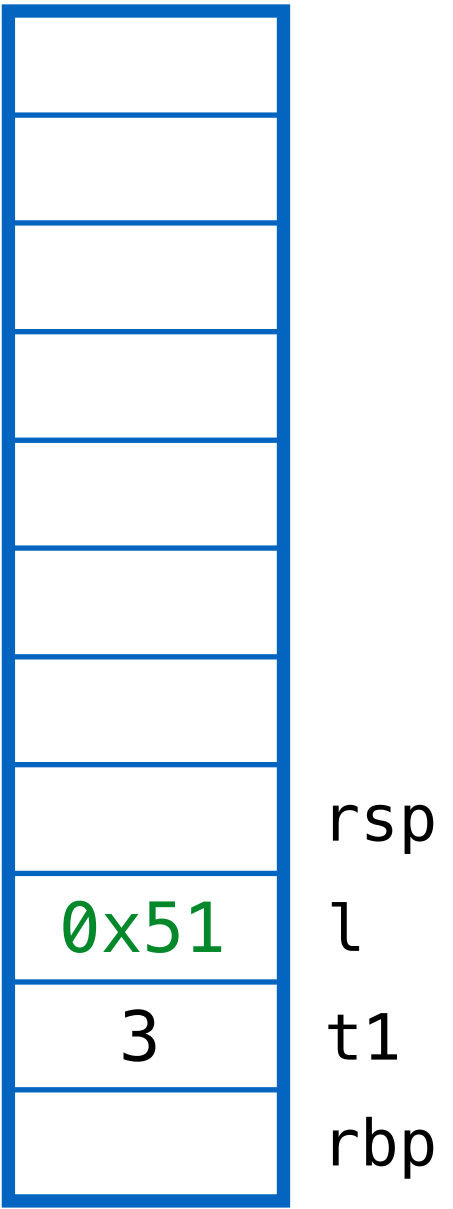
(E) (2, (1, (0, (3, (4, (5, false))))))



ex4: recursive data

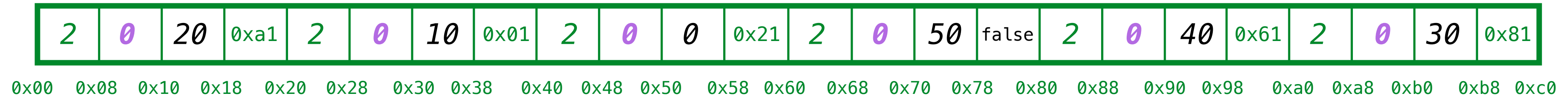
QUIZ: Which cells are "live" on the heap?

- (A) 0x00
- (B) 0x10
- (C) 0x20
- (D) 0x30
- (E) 0x40
- (F) 0x50



Heap

r15



2

Stack

